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Extensions of the Keen-Minsky Model for Financial Fragility

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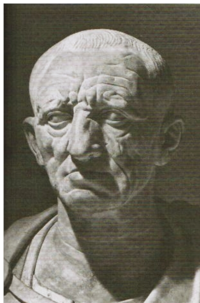
IMA, May 18, 2012

Why another talk on financial crises?

- Because they are a *hardy perennial*.
- Because macroeconomics is too important to be left at the hands of macroeconomists.
- Because *Carthago delenda est*



Dynamic Stochastic General Equilibrium

- Overwhelmingly dominant school in macroeconomics.
- Seeks to explain the aggregate economy using theories based on strong microeconomic foundations.
- All variables are assumed to be simultaneously in equilibrium.
- The only way the economy can be in disequilibrium at any point in time is through decisions based on wrong information.
- Money is neutral in its effect on real variables and only affects price levels.
- Largely ignores the role of irreducible uncertainty.

Hardcore (freshwater) DSGE

- The strand of DSGE economists affiliated with RBC theory made the following predictions after 2008:
  1. Increases government borrowing would lead to higher interest rates on government debt because of "crowding out".
  2. Increases in the money supply would lead to inflation.
  3. Fiscal stimulus has zero effect in an ideal world and negative effect in practice (because of decreased confidence).

Wrong prediction number 1

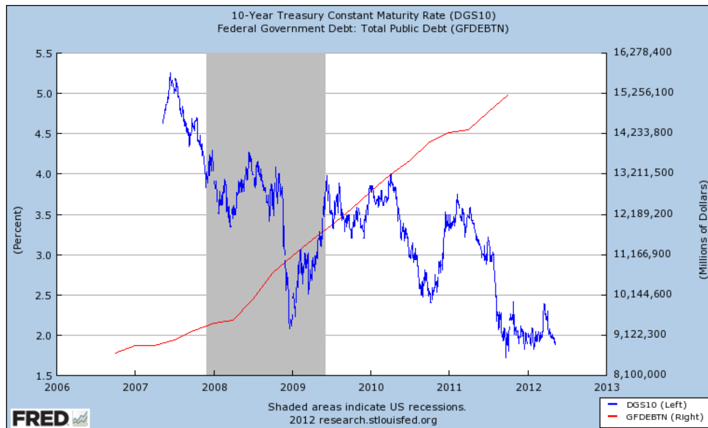


Figure: Government borrowing and interest rates.

Wrong prediction number 2

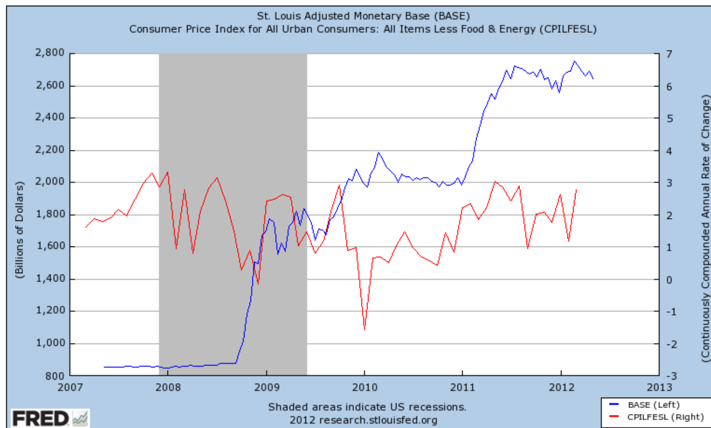


Figure: Monetary base and inflation.

Wrong prediction number 3

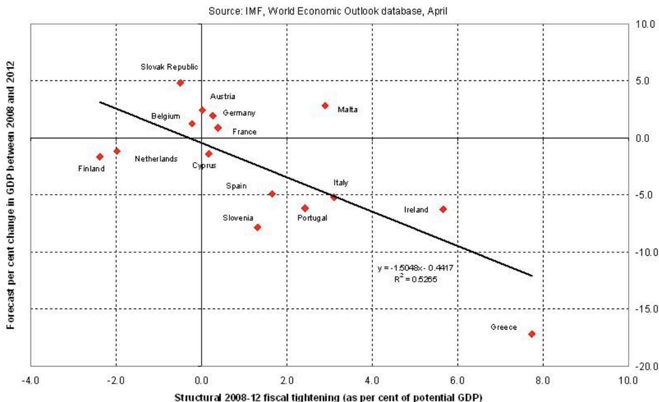


Figure: Fiscal tightening and GDP.

Meanwhile in Britain...

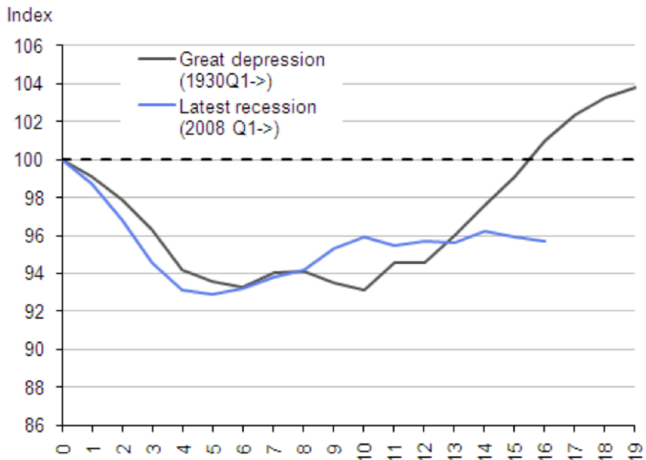


Figure: Office for National Statistics (UK), April 2012

Soft core (saltwater) DSGE

- The strand of DSGE economists affiliated with New Keynesian theory got all these predictions right.
- They did so by augmented DSGE with ‘imperfections’ (wage stickiness, asymmetric information, imperfect competition, etc).
- Still DSGE at core - analogous to adding epicycles to Ptolemaic planetary system.
- For example: “Ignoring the foreign component, or looking at the world as a whole, the overall level of debt makes no difference to aggregate net worth – one person’s liability is another person’s asset.” (Paul Krugman and Gauti B. Eggertsson, 2010, pp. 2-3)

Then we can safely ignore this...

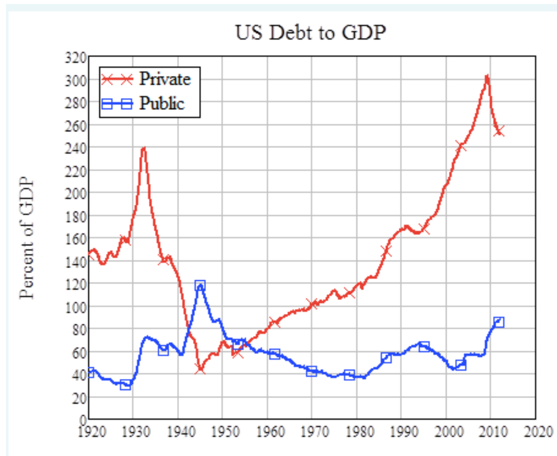


Figure: Private and public debt ratios.

Really?

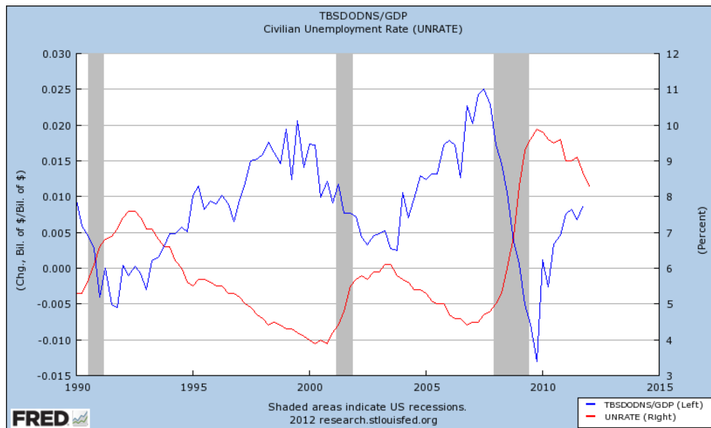


Figure: Change in debt and unemployment.

Minsky's alternative interpretation of Keynes

- Neoclassical economics is based on barter paradigm: money is convenient to eliminate the double coincidence of wants.
- In a modern economy, firms make complex portfolios decisions: which assets to hold and how to fund them.
- Financial institutions determine the way funds are available for ownership of capital and production.
- Uncertainty in valuation of cash flows (assets) and credit risk (liabilities) drive fluctuations in real demand and investment.
- Economy is fundamentally cyclical, with each state (boom, crisis, deflation, stagnation, expansion and recovery) containing the elements leading to the next in an identifiable manner.

Minsky's Financial Instability Hypothesis

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- Start when the economy is doing well but firms and banks are conservative.
- Most projects succeed - "Existing debt is easily validated: it pays to lever".
- Revised valuation of cash flows, exponential growth in credit, investment and asset prices.
- Highly liquid, low-yielding financial instruments are devalued, rise in corresponding interest rate.
- Beginning of "euphoric economy": increased debt to equity ratios, development of Ponzi financier.
- Viability of business activity is eventually compromised.
- Ponzi financiers have to sell assets, liquidity dries out, asset market is flooded.
- Euphoria becomes a panic.
- "Stability - or tranquility - in a world with a cyclical past and capitalist financial institutions is destabilizing".

Goodwin Model (1967) - Assumptions

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- Assume that

$$N(t) = N_0 e^{\beta t} \quad \text{(total labour force)}$$
$$a(t) = a_0 e^{\alpha t} \quad \text{(productivity per worker)}$$
$$Y(t) = \nu K(t) = a(t)L(t) \quad \text{(total yearly output)}$$

where K is the total stock of capital and L is the employed population.

- Assume further that

$$\dot{\mathrm{w}} = \Phi(\lambda)\mathrm{w} \quad \text{(Phillips curve)}$$
$$\dot{K} = (Y - \mathrm{w}L) - \delta K \quad \text{(Say's Law)}$$

Goodwin Model - Differential equations

- Define

$$\omega = \frac{\mathrm{w}L}{Y} = \frac{\mathrm{w}}{a} \quad \text{(wage share)}$$
$$\lambda = \frac{L}{N} = \frac{Y}{aN} \quad \text{(employment rate)}$$

- It then follows that

$$\dot{\omega} = \omega(\Phi(\lambda) - \alpha)$$
$$\dot{\lambda} = \lambda\left(\frac{1-\omega}{\nu} - \alpha - \beta - \delta\right)$$

Example 1: Goodwin model

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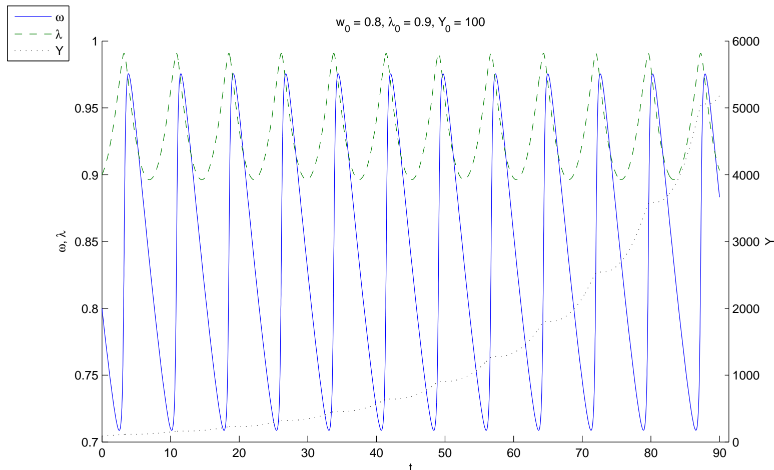
Example 1 (continued): Goodwin model



Testing Goodwin on OECD countries (Harvie 2000)

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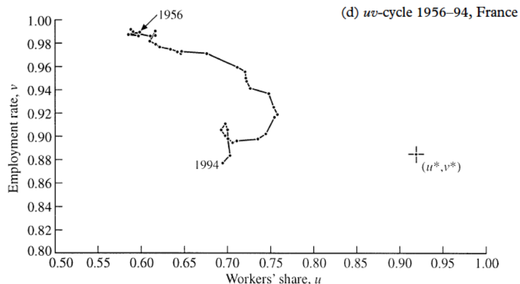

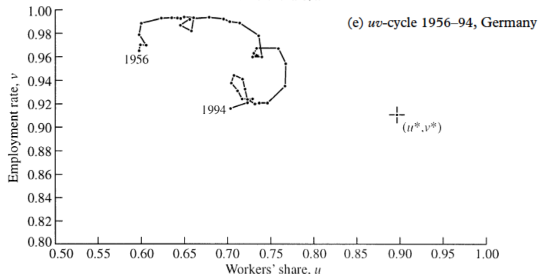

Introducing a financial sector (Keen 1995)

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- Assume now that new investment is given by

$$\dot{K} = \kappa(1 - \omega - rd)Y - \delta K$$

where $\kappa(\cdot)$ is $C^1(-\infty, \infty)$ increasing function satisfying certain technical conditions.

- Accordingly, total output evolves as

$$\frac{\dot{Y}}{Y} = \frac{\kappa(1 - \omega - rd)}{\nu} - \delta := g(\omega, d)$$

- This leads to external financing through debt evolving according to

$$\dot{D} = \kappa(1 - \omega - rd)Y - (1 - \omega - rd)Y$$

Keen model - Differential Equations

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Model with Noise

Denote the debt ratio in the economy by $d = D/Y$, the model can now be described by the following system

$$
\begin{aligned}
\dot{\omega} &= \omega \left[\Phi(\lambda) - \alpha\right] \\
\dot{\lambda} &= \lambda \left[\frac{\kappa(1-\omega-rd)}{\nu} - \alpha - \beta - \delta\right] \\
\dot{d} &= d \left[r - \frac{\kappa(1-\omega-rd)}{\nu} + \delta\right] + \kappa(1-\omega-rd) - (1-\omega)
\end{aligned}
\tag{1}
$$

Good equilibrium

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Model with Noise

- Define

$$\overline{\pi}_1 = \kappa^{-1}(\nu(\alpha+\beta+\delta))$$

- Then the following is an equilibrium for (1):

$$\overline{\omega}_1 = 1 - \overline{\pi}_1 - r\frac{\nu(\alpha+\beta+\delta)-\overline{\pi}_1}{\alpha+\beta}$$

$$\overline{\lambda}_1 = \Phi^{-1}(\alpha)$$

$$\overline{d}_1 = \frac{\nu(\alpha+\beta+\delta)-\overline{\pi}_1}{\alpha+\beta}$$

- Moreover

$$g(\overline{\omega}_1, \overline{d}_1) = \frac{\kappa(1-\overline{\omega}_1 - r\overline{d}_1)}{\nu} - \delta = \alpha+\beta.$$

Bad equilibrium

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Model with Noise

- If we rewrite the system with the change of variables $u = 1/d$, we obtain

$$\begin{aligned}
\dot{\omega} &= \omega\left[\Phi(\lambda) - \alpha\right] \\
\dot{\lambda} &= \lambda\left[\frac{\kappa(1-\omega-r/u)}{\nu} - \alpha - \beta - \delta\right] \\
\dot{u} &= u\left[\frac{\kappa(1-\omega-r/u)}{\nu} - r - \delta\right] - u^2\left[\kappa(1-\omega-r/u) - (1-\omega)\right].
\end{aligned} \tag{2}$$

- We now see that $(0,0,0)$ is an equilibrium of (2) corresponding to the point

$$(\overline{\omega}_2, \overline{\lambda}_2, \overline{d}_2) = (0, 0, +\infty)$$

for the original system.

Local stability

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- Analyzing the Jacobian of (1) and (2) we obtain the following conclusions.
- The good equilibrium $(\overline{\omega}_1, \overline{\lambda}_1, \overline{d}_1)$ is stable if and only if

$$r\left[\frac{\kappa'(\overline{\pi}_1)}{\nu}(\overline{\pi}_1 - \kappa(\overline{\pi}_1) + \nu(\alpha+\beta)) - (\alpha+\beta)\right] > 0.$$

- The point $(0,0,0)$ is a stable equilibrium for (2) if and only if

$$\frac{\kappa_0}{\nu} - \delta < r.$$

Example 2 : convergence to the good equilibrium in a Keen model

Example 2 (continued): convergence to the good equilibrium in a Keen model

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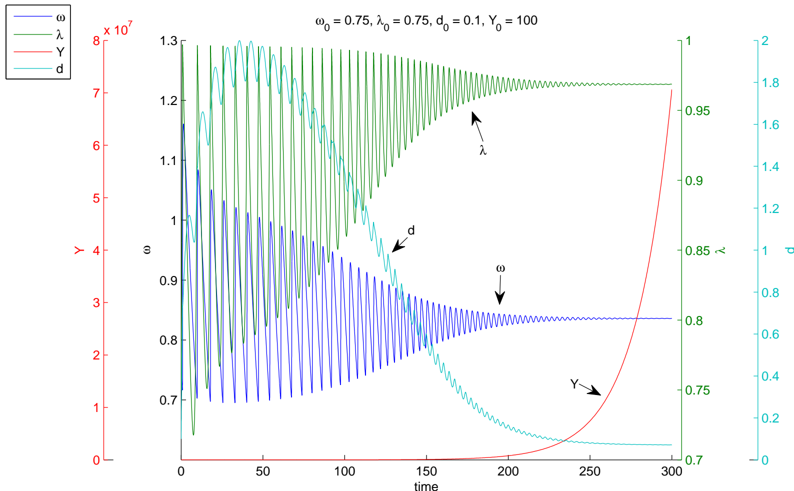
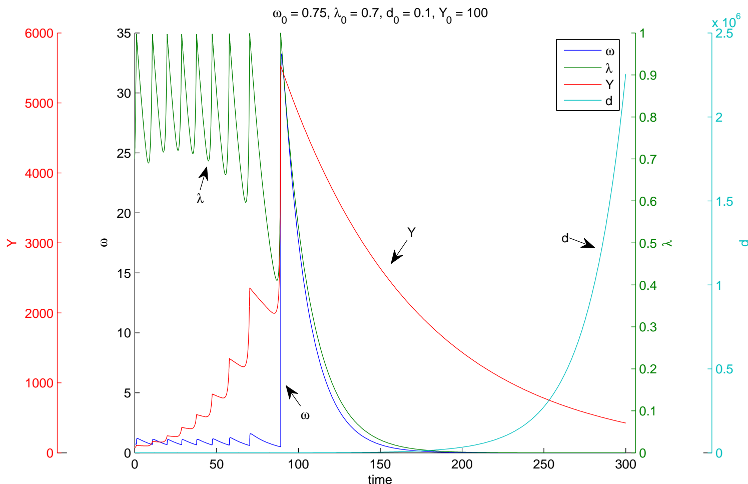


Example 3: explosive debt in a Keen model

Example 3 (continued): explosive debt in a Keen model



Example 3 (continued): explosive debt in a Keen model

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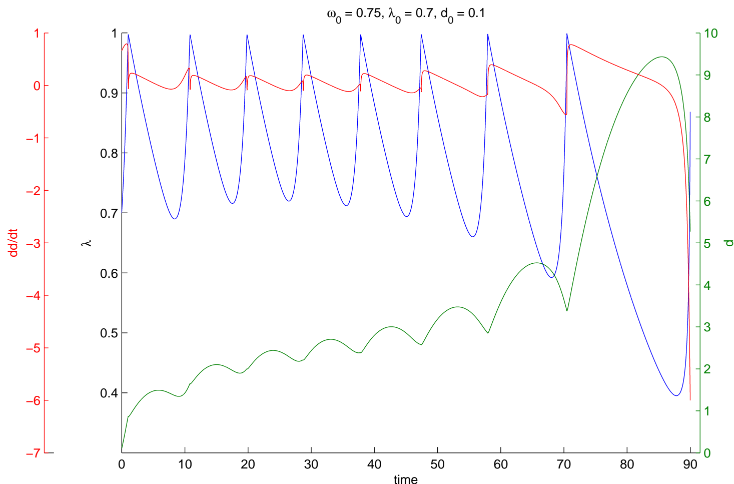
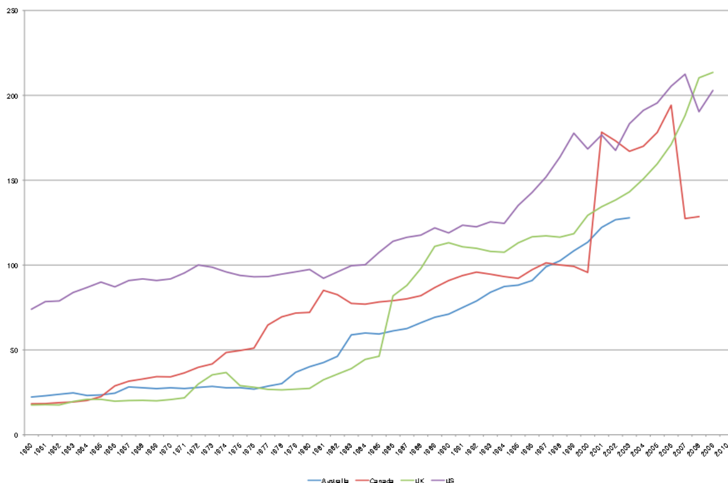
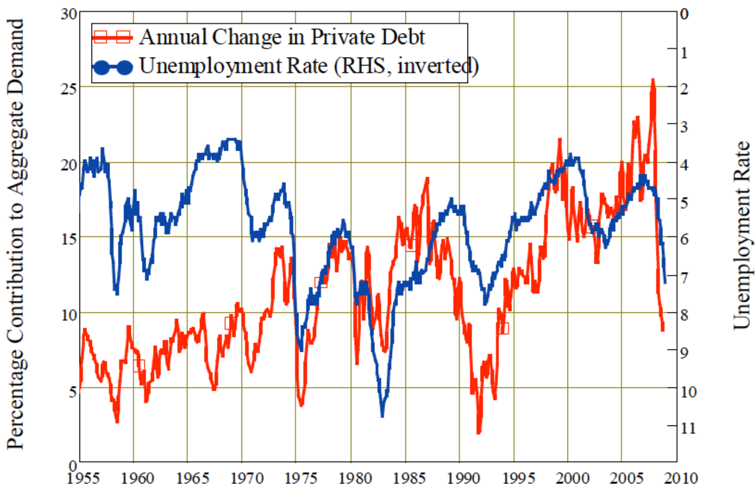
Example 3 (continued): explosive debt in a Keen model



Data detour: debt



Data detour: debt and employment



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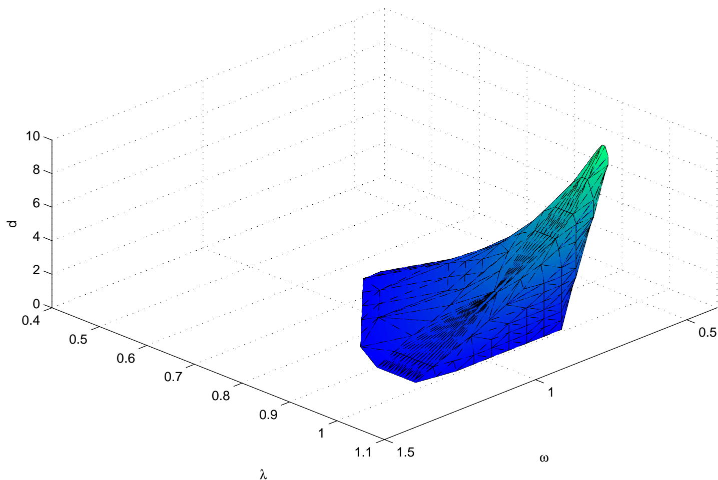
Basin of convergence for Keen model



Introducing a government sector

- Following Keen (and echoing Minsky) we add discretionary government spending and taxation into the original system according to

$$\dot{G} = \eta(\lambda)G$$
$$\dot{T} = \Theta(\pi)T$$

- Defining $g = G/Y$ and $t = T/Y$, the net profit share is now

$$\pi = 1 - \omega - rd + g - t,$$

and government debt evolves according to

$$\dot{D}_g = rD_g + G - T.$$

Example 4: Start with initial conditions near the locally stable equilibrium at infinite debt . . .

$\omega_0 = 0.75$, $\lambda_0 = 0.8$, $d_0 = 5$, $g_0 = 0$, $t_0 = 0$, $r = 0.03$, $\eta_{max} = 0.01$

d ($\times 10^{78}$): 0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5

ω: 0, 2, 4, 6, 8, 10, 12

λ: 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1

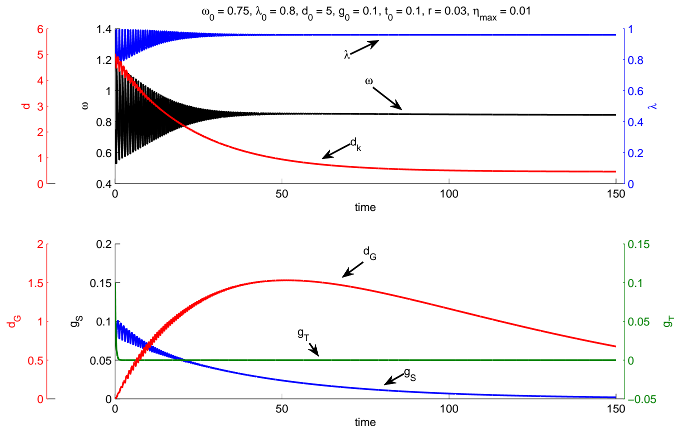
time: 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100

λ

ω

d_k

Example 4 (continued): ...then add government to drive it to the locally stable good equilibrium.



Example 4 (continued): But the system still crashes for sufficiently bad initial conditions!

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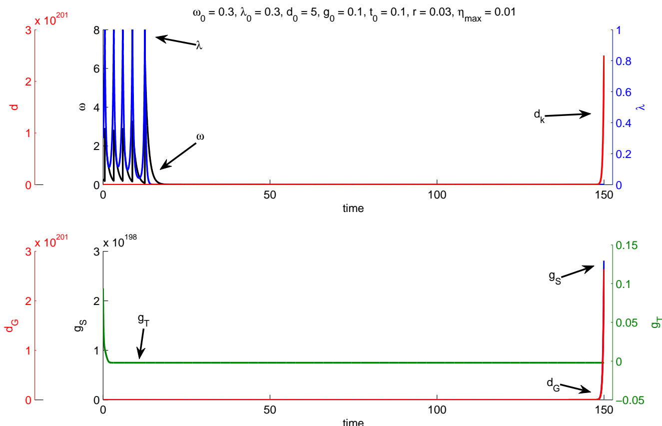
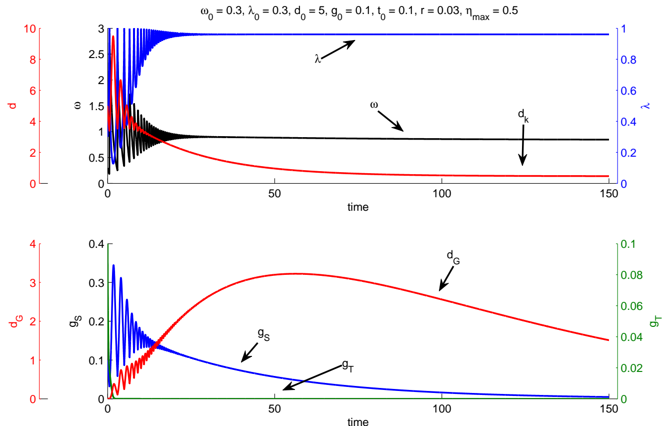
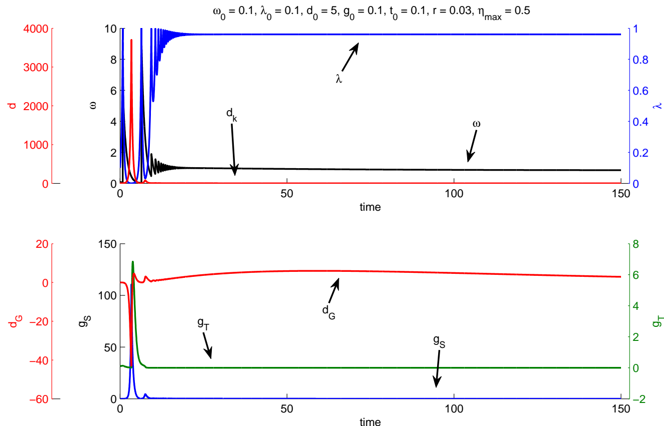


Example 6: Make government spending high enough, however, and the system is persistent ...



Example 6: ...no matter how bad it starts.



Ponzi financing

To introduce the destabilizing effect of purely speculative investment, we consider a modified version of the previous model with

$$\dot{D} = \kappa(1-\omega-rd)Y - (1-\omega-rd)Y + P$$
$$\dot{P} = \Psi(g(\omega,d)P$$

where $\Psi(\cdot)$ is an increasing function of the growth rate of economic output

$$g(\omega,d) = \frac{\kappa(1-\omega-rd)}{\nu} - \delta.$$

Example 4: effect of Ponzi financing

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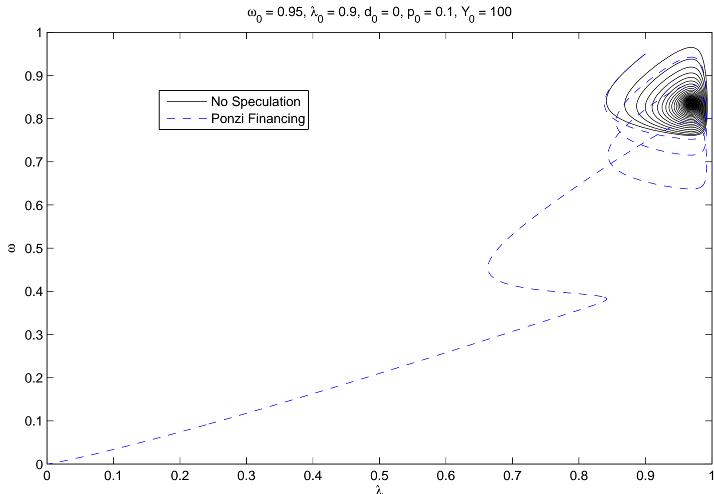
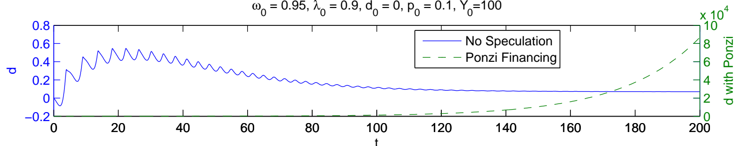
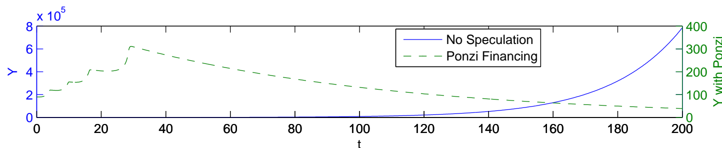
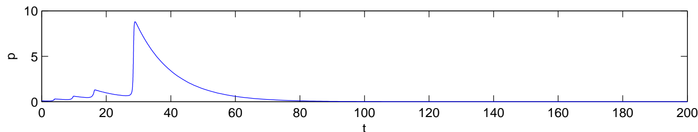


Example 4 (continued): effect of Ponzi financing







Stock prices

- Consider a stock price process of the form

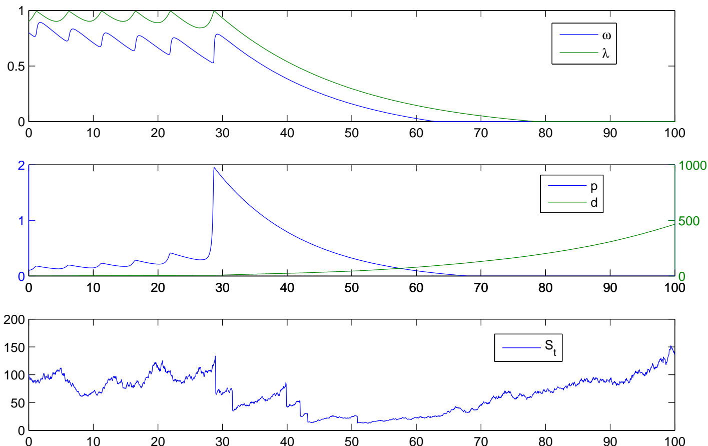
$$\frac{dS_t}{S_t} = r_b dt + \sigma dW_t + \gamma \mu_t dt - \gamma dN^{(\mu_t)}$$

where N_t is a Cox process with stochastic intensity $\mu_t = M(p(t))$.

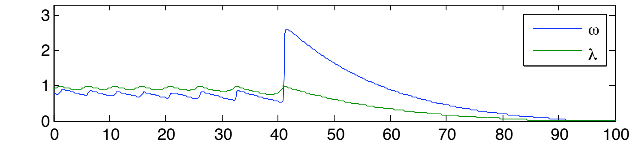
- The interest rate for private debt is modelled as $r_t = r_b + r_p(t)$ where

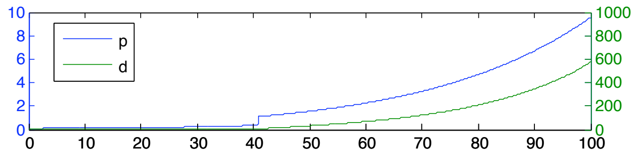
$$r_p(t) = \rho_1 (S_t + \rho_2)^{\rho_3}$$

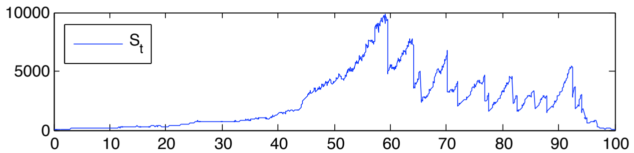
Example 6: stock prices, explosive debt, zero speculation



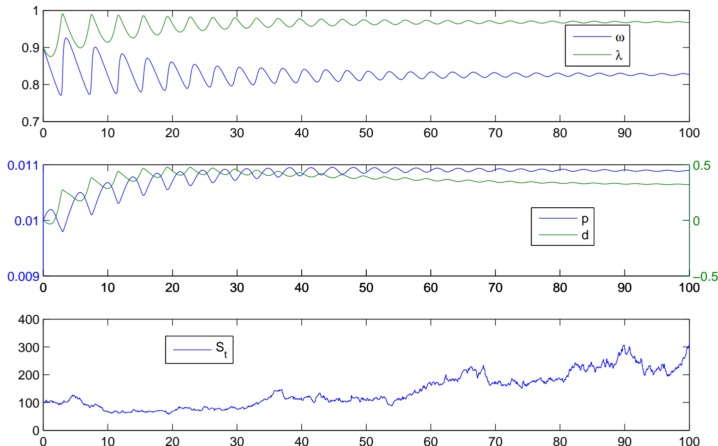
Example 6: stock prices, explosive debt, explosive speculation







Example 6: stock prices, finite debt, finite speculation



Stability map

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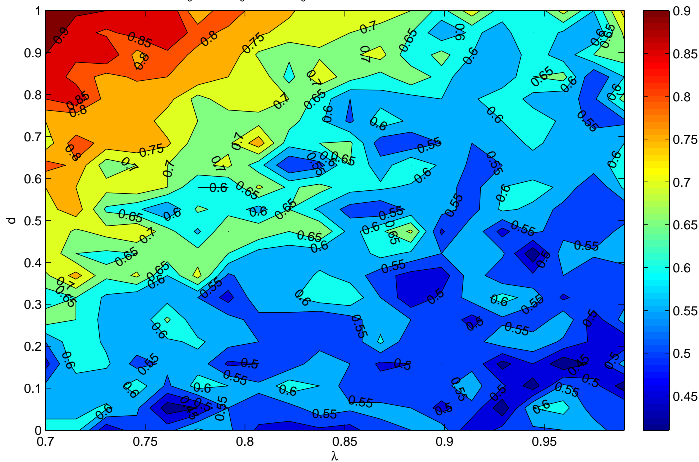

Next steps

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Model with Noise

- Introduce delay in the investment function
- Study stochastic model analytically
- Model prices for capital goods P_k and commodities P_c explicitly (Kaleckian mark-up theory, inflation, etc)
- Calibrate to macroeconomic time series.

Concluding thoughts

- Solow (1990): The true test of a simple model is whether it helps us to make sense of the world. Marx was, of course, dead wrong about this. We have changed the world in all sorts of ways, with mixed results; the point is to interpret it.
- Schumpeter (1939): Cycles are not, like tonsils, separable things that might be treated by themselves, but are, like the beat of the heart, of the essence of the organism that displays them.