

# An agent-based model for bank formation, bank runs and interbank networks

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An  
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# The quest to understand banking crises

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- Financial crises in the past 800 years encompass:
  - ① sovereign defaults
  - ② currency debasement and inflation
  - ③ exchange rate crises
  - ④ **banking crises**
- Graduating from banking crises has eluded developed and developing countries alike - Reinhart and Rogoff (2009).
- Individual banks are subject to runs, largely addressed by deposit insurance, capital requirements, and regulation.
- However, the principles that govern individual prudence do not necessarily apply to systems as a whole.
- Financial innovation and integration leads to highly interconnected, complex and potentially fragile banking systems.
- Systemic crises are essentially stories of contagion, interdependence, interaction and trust - Kirman (2010).

# Agent-Based Models in Economics

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- Modern macroeconomic theory (e.g 'sophisticated' DSGE models) is hopeless to deal with banking crises.
- Representative agents, neutrality of money, stationarity of expectations, and assumed equilibrium states are non-starters for the problem at hand.
- Agent-based computational economics (ACE) has emerged as an alternative.
- Agents have rational objectives, but realistic computational devices (inductive learning, bounded memory, limited information, war games, etc).
- Interactions are modelled directly, without fictitious clearing mechanisms.
- Hierarchical structures (i.e, banks are agents, but so are their clients, as well as the government).
- Equilibrium is just one possible outcome, not assumed a priori.

# Network paradigm

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- Focus on the relationships between different entities as well as the entities themselves
- Well suited to study systems where complexity arises from both the **interactions** among units and the **anatomy** of the system.
- Provide unifying principles for ecosystems, power transmission, infectious diseases, etc.
- In the context of banking, networks can help explain:
  - ① the effect of network structure on system stability
  - ② the dynamic evolution of interbank links in order to reduce exposure to risk
- The bulk of recent work on systemic risk focuses on the first aspect.

# Banking networks

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- Financial institutions are connected through:
  - ① direct links in the interbank market
  - ② indirect links through similar portfolio exposure
- Shocks come from assets or liabilities.
- For example, Allen and Gale (2000) investigate links of the first type and conclude that fully connected networks are robust to liquidity (liability) shocks.
- Alternatively, Cifuentes, Ferrucci and Shin (2005) consider exposure to common assets under market-to-market and minimal capital requirements and reach different conclusions.
- Unifying the effects of both types of links and shocks is still largely open.
- Most studies define failure as default and loss of capital.
- Systemic failure should also include cases where the network does not provide its social and economic function.

# Liquidity preferences

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- An asset is illiquid if its liquidation value at an earlier time is less than the present value of its future payoff.
- For example, an asset can pay  $1 \leq r_1 \leq r_2$  at dates  $T = 0, 1, 2$ .
- The lower the ratio  $r_1/r_2$  the less liquid is the asset.
- At time  $t = 0$ , consumers don't know in which future date they will consume.
- The consumer's expected utility is

$$wU(r_1) + (1 - w)U(r_2),$$

where  $w$  is the proportion of early consumers (type 1).

- Sufficiently risk-averse consumers prefer the liquid asset.

## Example: Diamond (2007)

- Let  $A = (r_1 = 1, r_2 = 2)$  represent an illiquid asset and  $B = (r_1 = 1.28, r_2 = 1.813)$  a liquid one.
- Assume investors with power utility  $u(c) = 1 - c^{-1}$  and  $w = 1/4$ .
- The expected utility from holding the illiquid asset is

$$E[u(c)] = \frac{1}{4}u(1) + \frac{3}{4}u(2) = 0.375$$

- By comparison, the expected utility from holding the liquid asset is

$$E[u(c)] = \frac{1}{4}u(1.28) + \frac{3}{4}u(1.813) = 0.391$$

- Observe, however, that risk-neutral investors would prefer the illiquid asset, since:

$$E[A] = 1.75 > 1.68 = E[B]$$



- Consider an economy with dates  $T = 0, 1, 2$  and an illiquid asset  $A = (1, R)$  and consumer preferences given by

$$U(c_1^j, c_2^j, \omega) = \begin{cases} u^j(c_1) & \text{if } j \text{ is of type 1 in state } \omega \\ u^j(c_2) & \text{if } j \text{ is of type 2 in state } \omega \end{cases} \quad (1)$$

- Denoting by  $w$  the fraction of early consumers (type 1), the optimal risk sharing for *publicly* observed preferences is

$$u'(c_1^{1*}) = Ru'(c_2^{2*}) \quad (2)$$

$$(1 - w)c_2^{2*} = (1 - wc_1^{1*})R \quad (3)$$

- However, liquidity preferences are private unverifiable information !

# A model for banks - Diamond and Dybvig (1983)

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- Suppose now that a bank offers a fixed claim  $r_1$  per unit deposited at time 0.
- Assume that withdrawers are served sequentially in random order until bank runs out of assets.
- Denoting by  $f_j$  the fraction of withdrawers before  $j$  and by  $f$  their total fraction, the payoffs per unit deposited are

$$V_1(f_j, r_1) = r_1 \mathbf{1}_{\{f_j < r_1^{-1}\}}$$

$$V_2(f, r_1) = [R(1 - r_1 f)/(1 - f)]^+$$

- Setting  $r_1 = c_1^{1*}$ , a good equilibrium corresponds to  $f = w$ , since this leads to  $V_2 = c_2^{2*} > c_1^{1*} = V_1$ .
- However, it is clear that  $f = 1$  (run) is also an equilibrium leading to  $V_1 \leq c_1^{1*}$  and  $V_2 = 0 < c_2^{2*}$ .

## Example revisited: Diamond (2007)

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- Let the illiquid asset be  $A = (1, 2)$ ,  $u(c) = 1 - c^{-1}$  and  $w = 1/4$
- Then the marginal utility condition becomes  $c_2^{2*} = \sqrt{R}c_1^{1*}$ .
- Substituting into the budget constraint (3) gives

$$c_1^{1*} = \frac{\sqrt{R}}{1 - w + w\sqrt{R}} = 1.28, \quad c_2^{2*} = 1.813.$$

- Suppose the bank offers the liquid asset  $B = (1.28, 1.813)$  to 100 depositors each with \$1 at 0 and invests in  $A$ .
- If  $f = 1/4$ , the bank needs to pay  $25 \times 1.28 = 32$  at  $t = 1$ .
- At  $t = 2$  the remaining depositors receive  $\frac{68 \times 2}{75} = 1.813$ .
- Therefore a forecast  $\hat{f} = 1/4$  is a Nash equilibrium.
- However, the forecast  $\hat{f} = 1$  is another Nash equilibrium.

# A model for interbank loans - Allen and Gale (2000)

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- Consider an economy with 4 banks (regions)  $A, B, C, D$ .
- There is a continuum of agents with unit endowment at time 0 and liquidity preferences given according to (1).
- The probability  $w$  of being an early consumer varies from one region to another conditional on two states  $S_1$  and  $S_2$  with equal probabilities:

Table: Regional Liquidity Shocks

	$A$	$B$	$C$	$D$
$S_1$	$w_H$	$w_L$	$w_H$	$w_L$
$S_2$	$w_L$	$w_H$	$w_L$	$w_H$

- Each bank can invest in a liquid asset  $(1, 1)$  and an illiquid asset  $(r < 1, R > 1)$  and promises consumption  $(c_1, c_2)$ .

# The central planner solution

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- The **central planner solution** consists of the best allocation  $(x, y)$  of per capita amounts invested in the illiquid and liquid assets maximizing the consumer's expected utility.

- This is easily seen to be given by

$$\gamma c_1 = y, \quad (1 - \gamma)c_2 = Rx,$$

where  $\gamma = \frac{w_H + w_L}{2}$  is the fraction of early consumers.

- Once liquidity is revealed, the central planner moves resources around.
- For example, in state  $S_1$ ,  $A$  and  $C$  have excess demand  $(w_H - \gamma)c_1$  at  $t = 1$ , which equals the excess supply  $(\gamma - w_L)c_1$  from  $B$  and  $D$ .
- At  $t = 2$  the flow is reversed, since the excess supply  $(w_H - \gamma)c_2$  from  $A$  and  $C$  equals the excess demand  $(\gamma - w_L)c_2$  from  $B$  and  $D$ .

# Optimal interbank loans

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- In the absence of a central planner, interbank loans can overcome the maldistribution of liquidity.
- Suppose that the network is completely connected (i.e. links between all banks).
- To achieve the optimal allocation, it is enough for banks to exchange deposits  $z_i = (w_H - \gamma)/2$  at time  $t = 0$ .
- At  $t = 1$ , a bank with high liquidity demand satisfies

$$\left[ w_H + \frac{w_H - \gamma}{2} \right] c_1 = y + \frac{3(w_H - \gamma)c_1}{2},$$

which reduces to  $\gamma c_1 = y$ .

- At  $t = 2$ , the same bank satisfies

$$[(1 - w_H) + (w_H - \gamma)]c_2 = Rx,$$

which reduces to  $(1 - \gamma)c_2 = Rx$ .

# Shocks and stability

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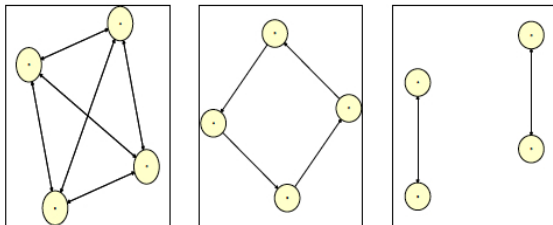
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- Allen and Gale (2000) then analyze the effects of small shocks to interbank markets with networks of the form:



- They show that the complete network absorbs shocks better than the incomplete one.
- Their analytic model is difficult to generalize to arbitrary (asymmetric) networks.

# Our model - the summarized story

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- Liquidity Preference
- Searching for partners
- Learning and Predicting
- Bank birth
- Interbank Links
- Contagion



- We have a society of individuals investing at the beginning of each period ( $t = 0$ ).
- For each individual  $i$ , an initial preference is drawn from a continuous uniform random variable  $U_i$
- If  $U_i < 0.5$  the agent is deemed to be liquid asset investor (short-term, early consumer), otherwise the agent is an illiquid asset investor (long-term, late consumer).
- There is a mid-period ( $t = 1$ ) shock to their preferences:

$$\tilde{U}_i = U_i + (-1)^{ran_i} \frac{\epsilon_i}{2}$$

- If  $\tilde{U}_i < 0.5$  the investor wants to be a short term investor, otherwise he wants to be long term investor.
- If the shock is big enough the individual wishes to have invested differently.
- Because of anticipated shocks, individuals explore the society searching to partners to exchange investments.

# Searching for partners

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- We impose some constraints on the individual capacity to go around and seek other individuals to trade.
- This reflects the inherited limited capability of information gathering and environment knowledge of individual agents.
- We use a combination of Von Neumann and Moore neighborhood:

5	1	6
2	X	3
7	4	8

# Inductive reasoning

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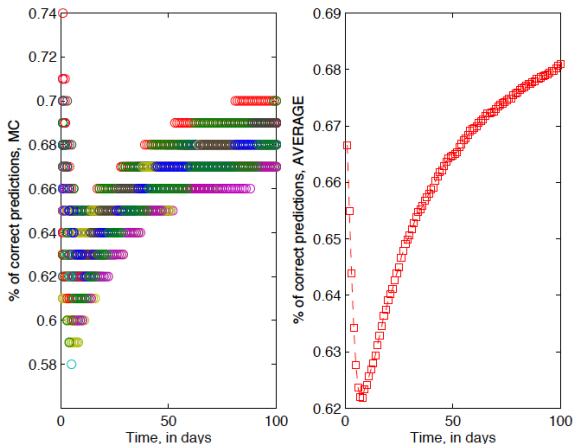
- We follow the inductive reasoning proposed by Arthur (2000) for individuals with bounded rationality dealing with complex environments.
- We assume agents make predictions using a memory of 5 periods.
- All agents have a set of 7 predictors as follows:
  - 1 Today would be the same as last period.
  - 2 Today would be the same as two periods ago.
  - 3 Today would be the same as three periods ago.
  - 4 Today would be the same as four periods ago.
  - 5 Today would be the same as five periods ago.
  - 6 Today would be the same as the mode for the last three periods.
  - 7 Today would be the same as the mode for the last five periods.

- Each predictor makes one of the following forecasts:
  - 1 N = agent will not need a partner
  - 2 G = agent will need a partner and will find one
  - 3 B = agent will need a partner and will not find one
- Depending on the realized outcome, a predictor's strength gets updated by

$$\Delta S = \begin{cases} +1 & \text{if the forecast is correct} \\ -1 & \text{if the forecast is incorrect} \end{cases}$$

# Learning simulation

We use 400 persons over a time span of 100 periods in a simulation with 100 realizations:



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# Bank birth

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- We follow the work of Howitt and Clower (1999, 2007) on the emergence of economic organizations.
- A randomly selected agent  $i$  is hit by the 'idea of entrepreneurship' and makes an initial estimate  $W^i = Z^i/8$  of the fraction of early consumers, where  $Z^i$  is a random integer in  $[0, 8]$  and reflects the entrepreneur's 'animal spirits'.
- The bank is established if there are  $x$  and  $y$  such that  $x + y \leq 1$  and

$$y = c_1 W_i$$

$$Rx = c_2(1 - W_i),$$

where  $(c_1, c_2)$  is the promised consumption.

- Individuals become aware of bank existence only if the bank lies in their neighbourhood
- In addition we give the bank the reach of its new members

# To join or not to join a bank

- Agents need to decide between trading directly either in the liquid asset  $(1, 1)$  or the illiquid asset  $(r < 1, R > 1)$  or joining the bank and receiving  $(c_1 > 1, c_2 < R)$ .
- For example, an agent who current has late preferences might have the following payoff table:

	forecast	strength	payoff (join)	payoff (not join)
1	N	-2	$c_2$	$R$
2	G	0	$c_1$	1
3	N	+1	$c_2$	$R$
4	B	-1	$c_1$	$r$
5	G	+1	$c_1$	1
6	N	0	$c_2$	$R$
7	B	+2	$c_1$	$r$

- The decision is based on the weighted sum of payoffs.

# Experiment: bank formation

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# Experiment (continued): established banks

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Day 100

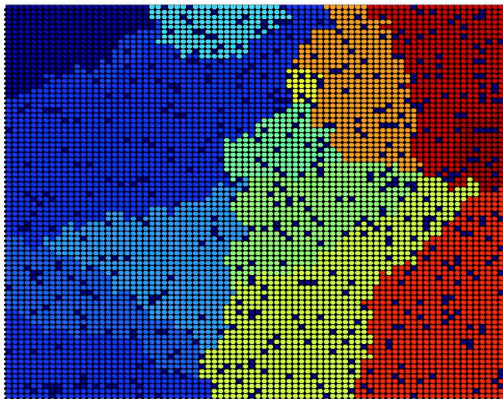


Figure: Banks at  $T=100$  with  $c_1 = 1.1$ ,  $c_2 = 1.5$  and  $R = 2$

# Experiment (continued): number of depositors

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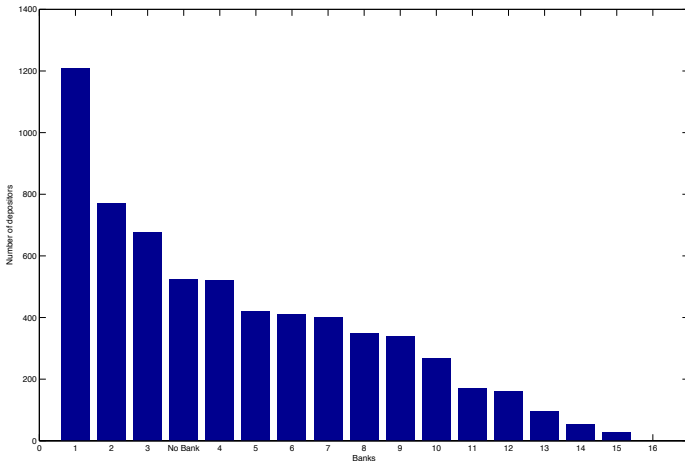
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- In the previous section we assumed that an agent never leaves a bank after joining.
- To model bank failures and runs we need a learning mechanism for banks themselves.
- Having made the allocation  $(x_t^i, y_t^i)$  based on  $W_t^i$ , banks accumulates reserves according to the realized  $\overline{W}_t^i$ :

$$C_t^i = [y_t^i - c_1 \overline{W}_t^i] + [Rx_t^i - c_2(1 - \overline{W}_t^i)].$$

- Banks update their estimate of early consumers through

$$W_{t+1}^i = \max \left\{ W_t^i + \alpha(\overline{W}_t^i - W_t^i), \frac{1 - c_2/R}{c_1 - c_2/R} \right\}, \quad (4)$$

reflecting both adaptation through a parameter  $\alpha \in (0, 1)$  and the budget constraint  $x_{t+1}^i + y_{t+1}^i \leq 1$  where

$$y_{t+1}^i = c_1 W_{t+1}^i, \quad Rx_{t+1}^i = c_2(1 - W_{t+1}^i).$$

# A run on the bank

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- We say that a bank is subject to a run if late consumers receive less than  $c_1$  at the end of the period.
- If the bank underestimates the fraction of early consumers, there is a run provided

$$(\bar{W}_t^i - W_t^i)c_1 > \left[ \frac{(1 - W_t^i)c_2}{R} - \frac{(1 - \bar{W}_t^i)c_1}{R} \right] r + C_t^i$$

- Conversely, if the bank overestimates  $\bar{W}_t^i$ , the amount available to late consumers (without using reserves) is

$$\begin{aligned} \frac{c_2(1 - W_t^i) + c_1(W_t^i - \bar{W}_t^i)}{1 - \bar{W}_t^i} &= c_2 - (c_2 - c_1) \frac{W_t^i - \bar{W}_t^i}{1 - \bar{W}_t^i} \\ &= c_1 + (c_2 - c_1) \frac{1 - W_t^i}{1 - \bar{W}_t^i} \end{aligned}$$

- The banks uses reserves to bring this as close as possible



# Experiment: bank formation and runs

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# Experiment: established banks (with possible runs)

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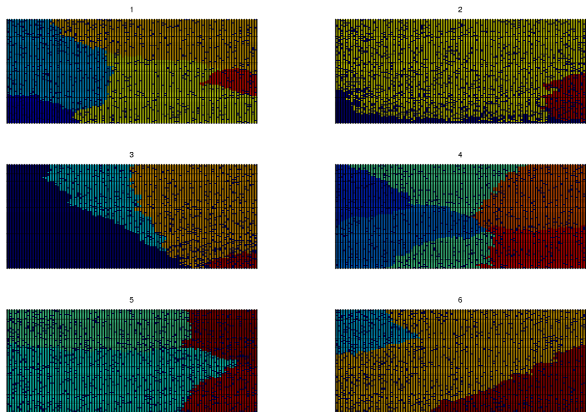


Figure: Banks at  $T=100$  with  $c_1 = 1.1$ ,  $c_2 = 1.5$  and  $R = 2$

# Banks and learning

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- As before, banks update their estimate of the fraction of early consumers according to (4).
- In addition, they deem the estimate to be adequate if the fraction of reserves lost in a given period is less than a certain threshold.
- They use the same set of predictors as clients to forecast the adequacy of their estimates as being 'adequate', 'inadequate' or 'undetermined'.
- Banks with inadequate or undetermined estimates have an incentive to exchange deposits with other banks and try to protect their reserves.

# Experiment: adequacy of estimates through time

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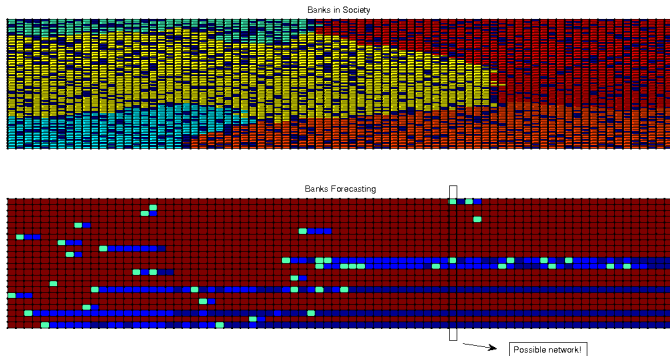
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**Figure:** Banks at  $T=100$  with  $c_1 = 1.1$ ,  $c_2 = 1.5$  and  $R = 2$  and adequacy of estimates over time.



# Experiment: possible network

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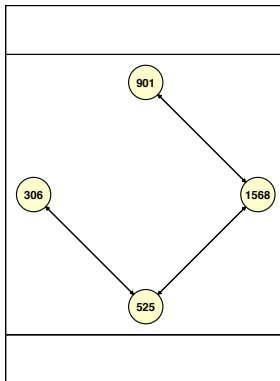


Figure: Snapshot of possible interbank loans

# Correlated liquidity shocks

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- As in Allen and Gale (2000), we consider regional liquidity shocks in a society with no overall shortage of liquidity.
- We form  $2C$  different regions (communities) as follows:
  - 1 Select  $2C$  cells at random to be the base
  - 2 Choose the largest reach  $M$  around the base
  - 3 Randomly select  $2M^2$  cells around the base to form a community
  - 4 Alter half of the communities to early preferences (i.e.  $\tilde{U}_i = 0.2$ ) and half of the communities to late preferences (i.e.  $\tilde{U}_i = 0.8$ ).

# Examples of correlated liquidity shocks

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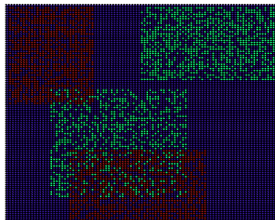
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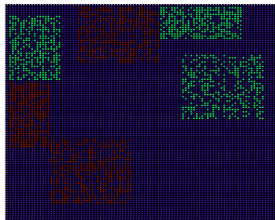
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Communities



Communities





# Experiment: bank formation and runs with correlated shocks

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# Experiment: adequacy of estimates through time (with correlated shocks)

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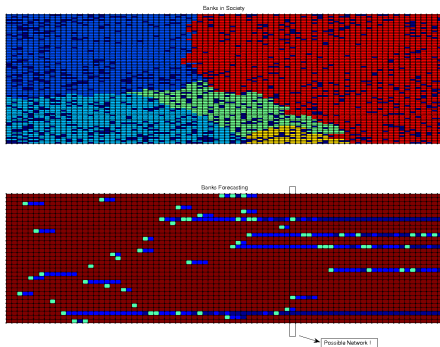
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**Figure:** Banks at  $T=100$  with  $c_1 = 1.1$ ,  $c_2 = 1.5$  and  $R = 2$  and adequacy of estimates over time.

# Experiment: another possible network

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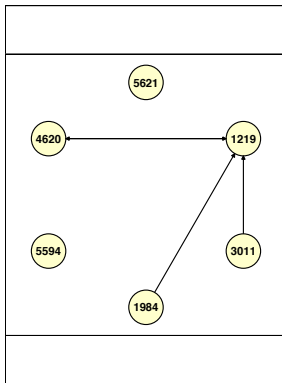
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**Figure:** Snapshot of possible interbank loans with correlated liquidity shocks

# Concluding remarks

An  
agent-based  
model for  
bank  
formation,  
bank runs and  
interbank  
networks

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Introduction

Theoretical  
underpinnings

Bank  
formation

Bank runs

Interbank  
networks

- We modelled individual liquidity preferences in a society.
- Changes in preferences lead agents to search for trading partners.
- Banks arise as providers of liquidity, but are inevitably subject to possible runs.
- Interbank loans redistributed the effect of correlated liquidity shocks across the society.
- Robustness of the model is being tested through extensive simulations.
- Ultimately want to adjust model parameters to reproduced different observed networks and use it as a testbed for policy implications.
- Thank you.