Austerity versus deficit spending: the mathematics of government intervention in macroeconomics

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What is wrong with this picture?

Figure: http://www.washingtonpost.com/wp-srv/special/politics/fiscal-cliff-questions/

What is the fiscal cliff?

Click on each question to expand or collapse an individual answer | Expand all | Collapse all |

1. What is the fiscal cliff? [–]

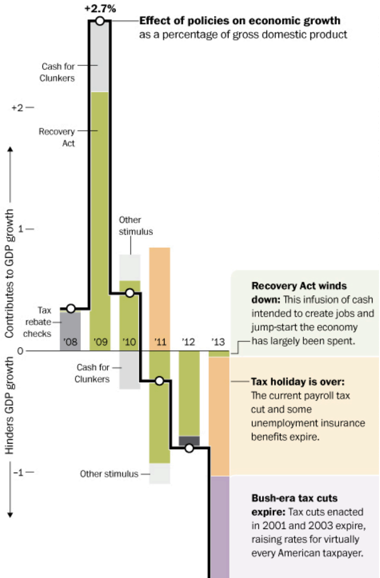
The term “fiscal cliff” is Washington shorthand for a series of automatic spending cuts and tax increases set to take effect in January. If enacted, they would amount to the largest spurt of deficit reduction in more than 40 years but could also push the country back into a recession.

Why is this a problem?

3. What happens if we go over the fiscal cliff?

Analysts have said that going over the fiscal cliff could derail the economy's fragile recovery. The nonpartisan Congressional Budget Office predicts that a recession would be significant but brief, with unemployment peaking around 9 percent and economic growth recovering during the second half of 2013. The International Monetary Fund has estimated that the automatic spending cuts and tax increases would knock perhaps four percentage points of growth off of a U.S. economy that is already only experiencing slow growth. Click on the image at right for a graphic explaining the situation.

In other words: Keynes roolz!



Really bad economics: hardcore (freshwater) DSGE

- The strand of DSGE economists affiliated with RBC theory made the following predictions after 2008:
  1. Increases government borrowing would lead to higher interest rates on government debt because of "crowding out".
  2. Increases in the money supply would lead to inflation.
  3. Fiscal stimulus has zero effect in an ideal world and negative effect in practice (because of decreased confidence).

Wrong prediction number 1

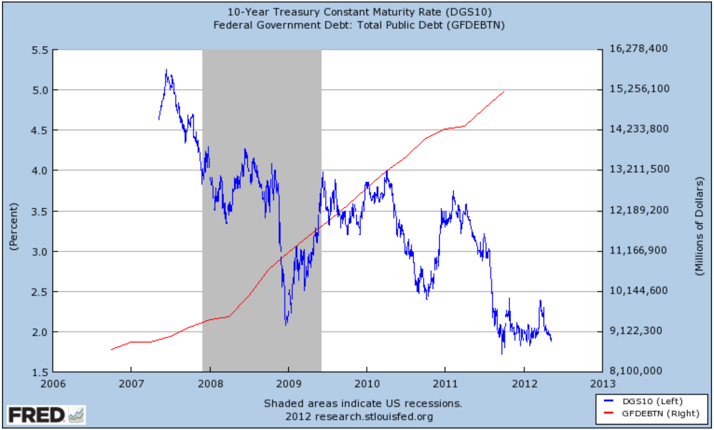


Figure: Government borrowing and interest rates.

Wrong prediction number 2

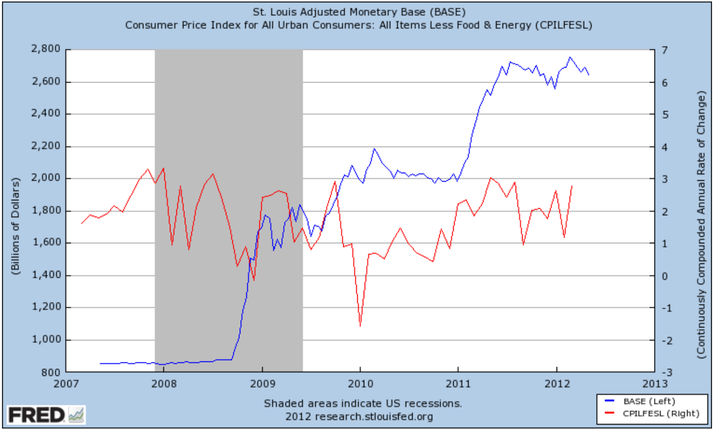


Figure: Monetary base and inflation.

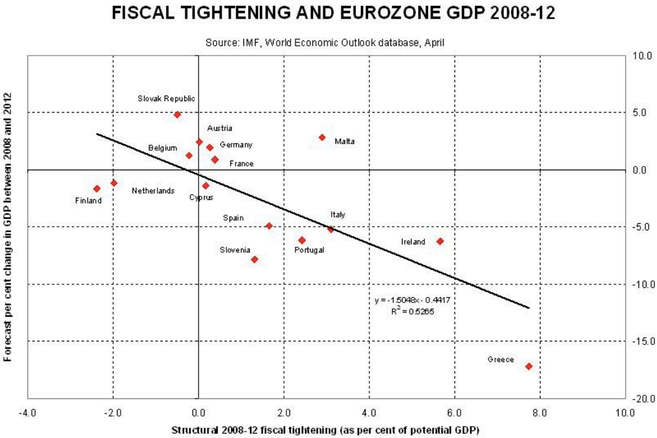


Figure: Fiscal tightening and GDP.

Better (but still bad) economics: soft core (saltwater) DSGE

- The strand of DSGE economists affiliated with New Keynesian theory got all these predictions right.
- They did so by augmented DSGE with 'imperfections' (wage stickiness, asymmetric information, imperfect competition, etc).
- Still DSGE at core - analogous to adding epicycles to Ptolemaic planetary system.
- For example: "Ignoring the foreign component, or looking at the world as a whole, the overall level of debt makes no difference to aggregate net worth – one person's liability is another person's asset." (Paul Krugman and Gauti B. Eggertsson, 2010, pp. 2-3)

Then we can safely ignore this...

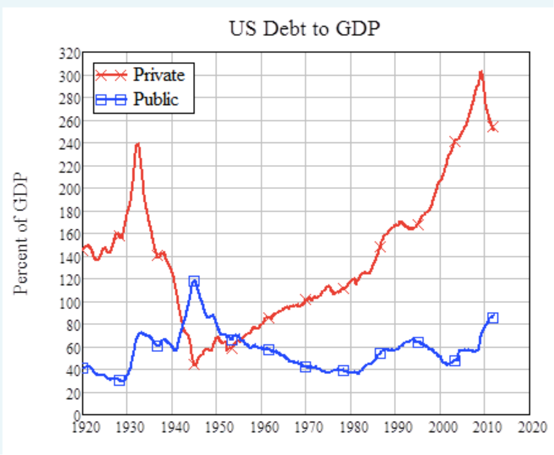


Figure: Private and public debt ratios.

Really?

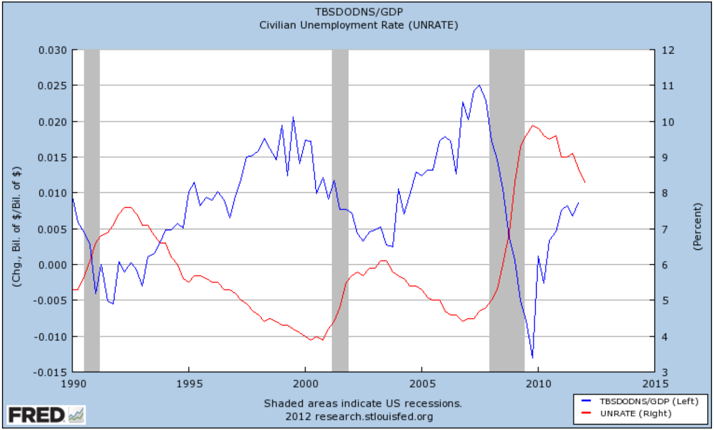


Figure: Change in debt and unemployment.

Much better economics: SFC models

- Stock-flow consistent models emerged in the last decade as a common language for many heterodox schools of thought in economics.
- Consider both real and monetary factors from the start
- Specify the balance sheet and transactions between sectors
- Accommodate a number of behavioural assumptions in a way that is consistent with the underlying accounting structure.
- Reject silly (and mathematically unsound!) hypotheses such as the RARE individual (representative agent with rational expectations).
- See Godley and Lavoie (2007) for the full framework.

An example of a (fairly general) Godley table

Balance Sheet	Households	Firms current	Firms capital	Banks	Central Bank	Gov	Sum
Capital			$+K$				$+K$
Cash	$+H_h$		$+H_f$	$+H_b$	$-H$		0
Advances				$-A$	$+A$		0
Deposits	$+M_h$		$+M_f$	$-M$			0
Loans			$-L$	$+L$			0
Bills	$+B_h$		$+B_f$	$+B_b$	$+B_c$	$-B$	0
Equities	$+p_f E_f + p_b E_b$		$-p_f E_f$	$-p_b E_b$			0
Sum (net worth)	V_h		V_f	V_b	0	V_g	K
Transactions							
Consumption	$-C$	$+C$					0
Gov spending		$+G$				$-G$	0
Investment		$+I$	$-I$				0
memo [GDP]		[Y]					
Wages	$+W$	$-W$					0
Taxes	$-T_h$	$-T_f$		$-T_b$		$+T$	0
Interest on deposits	$+r_M M_h$	$+r_M M_f$		$-r_M M$			0
Interest on loans		$-r_L L$		$+r_L L - r_A A$	$+r_A A$		0
Interest on bills	$+r_B B_h$	$+r_B B_f$		$+r_B B_b$	$+r_B B_c$	$-r_B B$	0
Profits	$+F_{fd} + F_b$	$-F_f$	$+F_{fu}$	$-F_b$	$-F_c$	$+F_c$	0
Financial Balances	S_h	0	S_f	S_b	0	S_g	0
Flow of Funds							
Cash	$-\dot{H}_h$		$-\dot{H}_f$	$-\dot{H}_b$	$+\dot{H}$		0
Advances				$+\dot{A}$	$-\dot{A}$		0
Deposits	$-\dot{M}_h$		$-\dot{M}_f$	$+\dot{M}$			0
Loans			$+\dot{L}$	$-\dot{L}$			0
Bills	$-\dot{B}_h$		$-\dot{B}_f$	$-\dot{B}_b$	$-\dot{B}_c$	$+\dot{B}$	0
Equities	$-p_f \dot{E}_f - p_b \dot{E}_b$		$+p_f \dot{E}_f$	$+p_b \dot{E}_b$			0
Column sum	0	0	0	0	0	0	0

Balance Sheet	Households	Firms current	Firms capital	Banks	Sum
Capital goods			$+K$		$+K$
Deposits	$+M_h$		$+M_f$	$-M$	0
Loans			$-L$	$+L$	0
Sum (net worth)	V_h		V_f	V_b	$+K$
Transactions					
Consumption	$-C$	$+C$			0
Investment		$+I$	$-I$		0
Accounting memo [GDP]		[Y]			
Wages	$+W$	$-W$			0
Interest on M	$+r_M M_h$	$+r_M M_f$		$-r_M M$	0
Interest on L		$-r_L L$		$+r_L L$	0
Profits		$-F_f$	$+F_{fu}$		0
Financial Balances	S_h	0	S_f	S_b	0
Flow of Funds					
Deposits	$-\dot{M}_h$		$-\dot{M}_f$	$+\dot{M}$	0
Loans			$+\dot{L}$	$-\dot{L}$	0
Column sum	0	0	0	0	0

Accounting relations for monetary Keen model

- From the first, second and third columns of the transactions and flow of funds matrices we obtain that

$$W + r_M M_h - C = S_h = \dot{M}_h, \tag{1}$$

$$C - W + r_M M_f - r_L L = F_{fu} - I = \dot{M}_f - \dot{L} \tag{2}$$

$$r_L L - r_M M = S_b = \dot{L} - \dot{M}. \tag{3}$$

- An example of firm behaviour satisfying (1)–(3) is

$$\dot{L} = I - R \tag{4}$$

$$\dot{M}_f = I - R + C - W + r_M M_f - r_L L \tag{5}$$

for chosen levels of investment I and repayment R.

Behavioral relationships

- Investment and repayment can then be modelled as a function of any firm-related variable:

$$I = \kappa(\omega, L, M_f), \tag{6}$$

$$R = \rho(\omega, L, M_f). \tag{7}$$

- Consumption is a function of household-related variables:

$$C = \xi(\omega, M_h). \tag{8}$$

- Wage changes are given by

$$\dot{\mathrm{w}} = \Phi(\omega, \lambda, p). \tag{9}$$

- Finally, the price level evolves according to

$$\dot{p} = P(\omega, \lambda, p). \tag{10}$$

Differential Equations

Collecting these definitions and that $K_R = \nu Y_R$ for a constant ν, we arrive at the following closed 6-dimensional system

$$
\begin{aligned}
\dot{\mathrm{w}} &= \Phi(\omega, \lambda, p) \\
\dot{\lambda} &= \lambda \left[\frac{\kappa(\omega, L, M_f)}{\nu} - \alpha - \beta - \delta \right] \\
\dot{p} &= P(\omega, \lambda, p) \\
\dot{L} &= \kappa(\omega, L, M_f) Y - \rho(\omega, L, M_f) \\
\dot{M}_f &= \kappa(\omega, L, M_f) Y - \rho(\omega, L, M_f) \\
&\quad + \xi(\omega, M_h) - \mathrm{w}\ell + r_M M_f - r_L L \\
\dot{M}_h &= \mathrm{w}\ell + r_M M_h - \xi(\omega, M_h),
\end{aligned}
\tag{11}
$$

where $\ell = \lambda N$ and $Y = a\ell$, for total population N and productivity a.

Special case: Keen (1995)

- Let $D = L - M_f$ and assume that $p = p_0$, $r_M = r_F = r$.
- Supposing further that $\Phi = \Phi(\lambda)$ and $I = \kappa(\pi)Y$, where $\pi = 1 - \omega - rd$, leads to

$$
\begin{aligned}
\dot{\omega} &= \omega\left[\Phi(\lambda) - \alpha\right] \\
\dot{\lambda} &= \lambda\left[\frac{\kappa(1-\omega-rd)}{\nu} - \alpha - \beta - \delta\right] \qquad (12) \\
\dot{d} &= d\left[r - \frac{\kappa(1-\omega-rd)}{\nu} + \delta\right] + \kappa(1-\omega-rd) - (1-\omega)
\end{aligned}
$$

- Observe that the equation for M_h separates as

$$\dot{M}_h = \mathrm{w}\ell + r_M M_h - \xi(\omega, M_h), \qquad (13)$$

and only depends on the rest of the system through w.

Equilibria

- The system (12) has a good equilibrium at

$$\overline{\omega} = 1 - \overline{\pi} - r\frac{\nu(\alpha+\beta+\delta) - \overline{\pi}}{\alpha+\beta}$$

$$\overline{\lambda} = \Phi^{-1}(\alpha)$$

$$\overline{d} = \frac{\nu(\alpha+\beta+\delta) - \overline{\pi}}{\alpha+\beta}$$

with

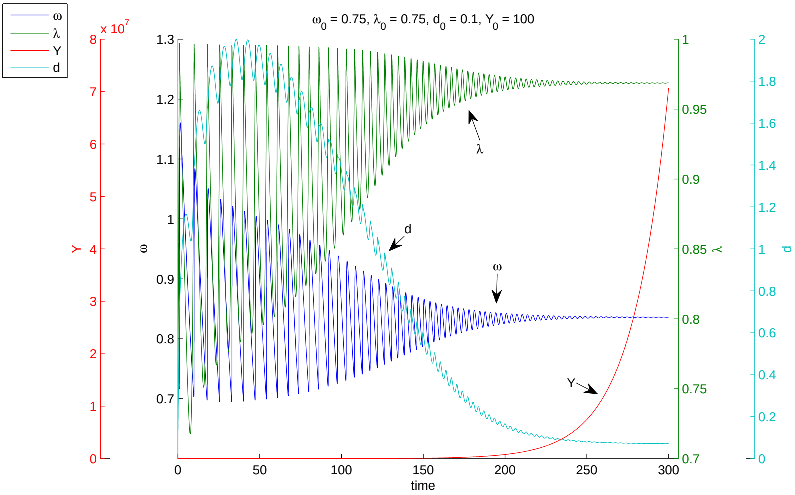
$$\overline{\pi} = \kappa^{-1}(\nu(\alpha+\beta+\delta)),$$

which is stable for a large range of parameters

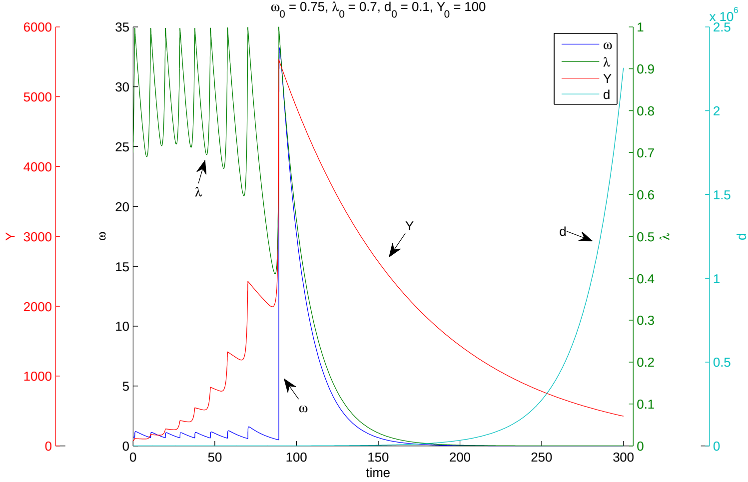
- It also has a bad equilibrium at $(0, 0, +\infty)$, which is stable if

$$\frac{\kappa(-\infty)}{\nu} - \delta < r \tag{14}$$

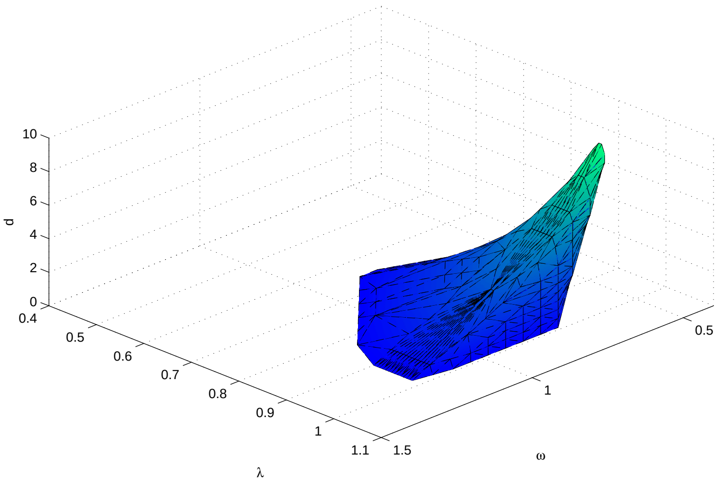
Example 1: convergence to the good equilibrium in a Keen model



Example 2: explosive debt in a Keen model



Basin of convergence for Keen model



Persistence theory on mathematical biology

- Which species, in a mathematical model of interacting species, will survive over the long term?
- In a mathematical model of an epidemic, will the disease drive a host population to extinction or will the host persist?
- Can a disease remain endemic in a population?
- Reference: H. Smith and H. R. Thieme, *Dynamical Systems and Population Persistence*, Graduate Studies in Mathematics, 118. AMS, 2011.

Persistence definitions

Let $\Phi(t,x) : \mathbb{R}^+ \times X \to X$ be the semiflow generated by a differential system with initial values $x \in X$. For a nonnegative functional ρ from X to $\mathbb{R}^+$, we say

- Φ is ρ–**strongly persistent** (SP) if $\liminf_{t\to\infty} \rho(\Phi(t,x)) > 0$ for any $x \in X$ with $\rho(x) > 0$.
- Φ is ρ–**weakly persistent** (WP) if $\limsup_{t\to\infty} \rho(\Phi(t,x)) > 0$ for any $x \in X$ with $\rho(x) > 0$.
- Φ is ρ–**uniformly strongly persistent** (USP) if $\liminf_{t\to\infty} \rho(\Phi(t,x)) > \varepsilon$ for any $x \in X$ with $\rho(x) > 0$.
- Φ is ρ–**uniformly weakly persistent** (UWP) if $\limsup_{t\to\infty} \rho(\Phi(t,x)) > \varepsilon$ for any $x \in X$ with $\rho(x) > 0$.

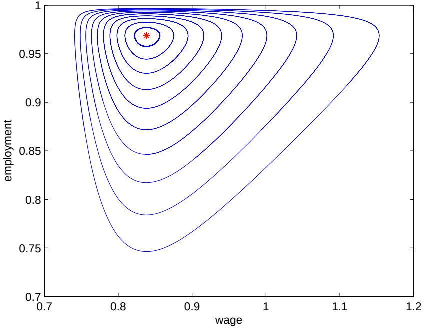
Example: Goodwin model

- (Goodwin 1967) Predator-prey system of wage share (ω) and employment rate (λ), with $\pi = 1 - \omega$:

$$\dot{\omega} = \omega[\Phi(\lambda) - \alpha]$$

$$\dot{\lambda} = \lambda[\pi/\nu - \alpha - \beta - \delta].$$

- This is e^{π}–SP and e^{π}–UWP, but not e^{π}–USP.



Introducing a government sector

- Following Keen (and echoing Minsky) we add discretionary government spending and taxation into the original system in the form

$$G = G_1 + G_2$$
$$T = T_1 + T_2$$

where

$$\dot{G}_1 = \eta_1(\lambda)Y \qquad \dot{G}_2 = \eta_2(\lambda)G_2$$
$$\dot{T}_1 = \Theta_1(\pi)Y \qquad \dot{T}_2 = \Theta_2(\pi)T_2$$

- Defining $g = G/Y$ and $\tau = T/Y$, the net profit share is now

$$\pi = 1 - \omega - rd + g - \tau,$$

and government debt evolves according to

$$\dot{B} = rB + G - T.$$

Differential equations - full system

Denoting $\gamma(\pi) = \kappa(\pi)/\nu - \delta$, a bit of algebra leads to the following eight–dimensional system:

$$\begin{aligned}
\dot{\omega} &= \omega[\Phi(\lambda) - \alpha] \\
\dot{\lambda} &= \lambda[\gamma(\pi) - \alpha - \beta] \\
\dot{d} &= \kappa(\pi) - \pi - d\gamma(\pi) \\
\dot{g}_1 &= \eta_1(\lambda) - g_1\gamma(\pi) \\
\dot{g}_2 &= g_2\left[\eta_2(\lambda) - \gamma(\pi)\right] \\
\dot{\tau}_1 &= \Theta_1(\pi) - g_{\tau_1}\gamma(\pi) \\
\dot{\tau}_2 &= \tau_2\left[\Theta_2(\pi) - \gamma(\pi)\right] \\
\dot{b} &= b[r - \gamma(\pi)] + g_1 + g_2 - \tau_1 - \tau_2
\end{aligned} \tag{15}$$

Differential equations - reduced system

- Notice that π does not depend on b, so that the last equation in (15) can be solved separately.
- Observe further that we can write

$$\dot{\pi} = -\dot{\omega} - r\dot{d} + \dot{g} - \dot{\tau} \tag{16}$$

leading to the five-dimensional system

$$\begin{aligned}
\dot{\omega} &= \omega\left[\Phi(\lambda) - \alpha\right], \\
\dot{\lambda} &= \lambda\left[\gamma(\pi) - \alpha - \beta\right] \\
\dot{g}_2 &= g_2\left[\eta_2(\lambda) - \gamma(\pi)\right] \\
\dot{\tau}_2 &= \tau_2\left[\Theta_2(\pi) - \gamma(\pi)\right] \\
\dot{\pi} &= -\omega(\Phi(\lambda) - \alpha) - r(\kappa(\pi) - \pi) + (1 - \omega - \pi)\gamma(\pi) \\
&\quad + \eta_1(\lambda) + g_2\eta_2(\lambda) - \Theta_2(\pi) - \tau_2\Theta_2(\pi)
\end{aligned} \tag{17}$$

Good equilibrium

- The system (17) has a good equilibrium at

$$
\begin{aligned}
\overline{\omega} &= 1 - \overline{\pi} - r\frac{\nu(\alpha+\beta+\delta) - \overline{\pi}}{\alpha+\beta} + \frac{\eta_1(\overline{\lambda}) - \Theta_1(\overline{\pi})}{\alpha+\beta} \\
\overline{\lambda} &= \Phi^{-1}(\alpha) \\
\overline{\pi} &= \kappa^{-1}(\nu(\alpha+\beta+\delta)) \\
\overline{g}_2 &= \overline{\tau}_2 = 0
\end{aligned}
$$

  and this is locally stable for a large range of parameters.
- The other variables then converge exponentially fast to

$$
\begin{aligned}
\overline{d} &= \frac{\nu(\alpha+\beta+\delta) - \overline{\pi}}{\alpha+\beta} \\
\overline{g}_1 &= \frac{\eta_1(\overline{\lambda})}{\alpha+\beta} \\
\overline{\tau}_1 &= \frac{\Theta_1(\overline{\pi})}{\alpha+\beta}
\end{aligned}
$$

Bad equilibria - destabilizing a stable crisis

- Recall that $\pi = 1 - \omega - rd + g - \tau$.
- The system (17) has bad equilibria of the form

$$
\begin{aligned}
(\omega, \lambda, g_2, \tau_2, \pi) &= (0, 0, 0, 0, -\infty) \\
(\omega, \lambda, g_2, \tau_2, \pi) &= (0, 0, \pm\infty, 0, -\infty)
\end{aligned}
$$

- If $g_2(0) > 0$, then any equilibria with $\pi \to -\infty$ is locally unstable provided $\eta_2(0) > r$.
- On the other hand, if $g_2(0) < 0$ (austerity), then these equilibria are all locally stable.

Persistence results

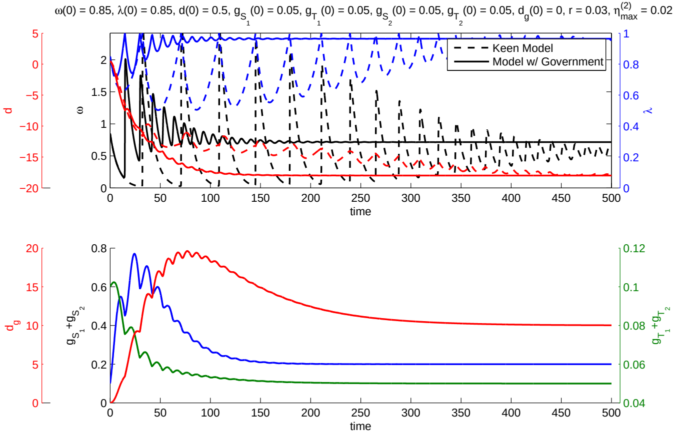
Proposition 1: Assume $g_2(0) > 0$, then the system (17) is e^π-UWP if either

1. $\lambda\eta_1(\lambda)$ is bounded below as $\lambda \to 0$, or
2. $\eta_2(0) > r$.

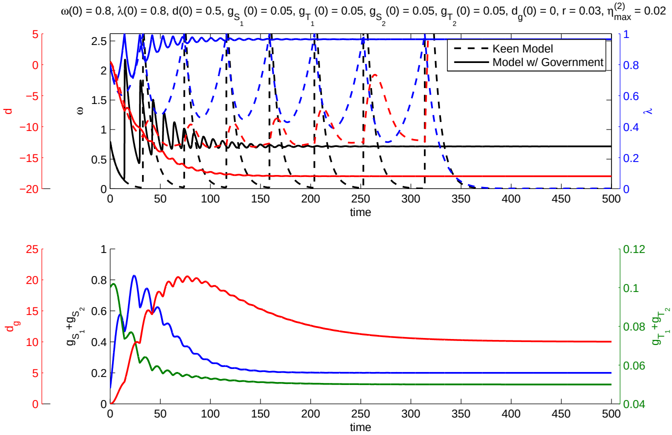
Proposition 2: Assume $g_2(0) > 0$ and $\tau_2(0) = 0$, then the system (17) is λ-UWP if either of the following three conditions is satisfied:

1. $\lambda\eta_1(\lambda)$ is bounded below as $\lambda \to 0$, or
2. $\eta_2(0) > \max\{r, \alpha + \beta\}$, or
3. $r < \eta_2(0) \le \alpha + \beta$ and $-r(\kappa(x) - x) + (1 - x)\gamma(x) + \eta_1(0) - \Theta_1(x) > 0$ for $\gamma(x) \in [\eta_2(0), \alpha + \beta]$.

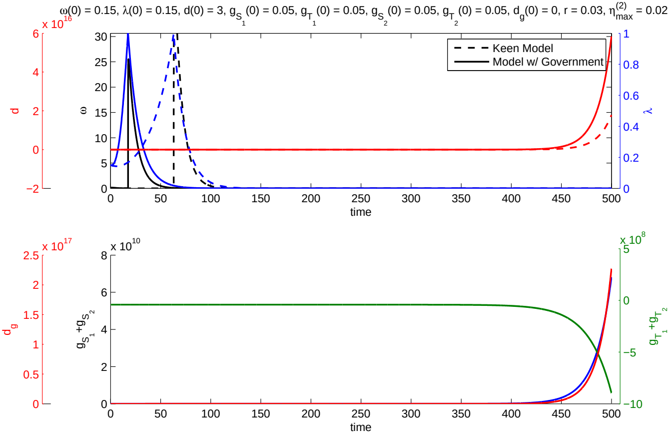
Example 3: Good initial conditions



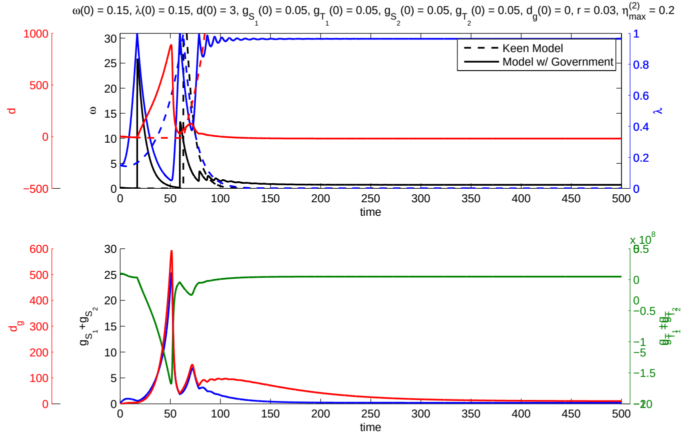
Example 4: Bad initial conditions



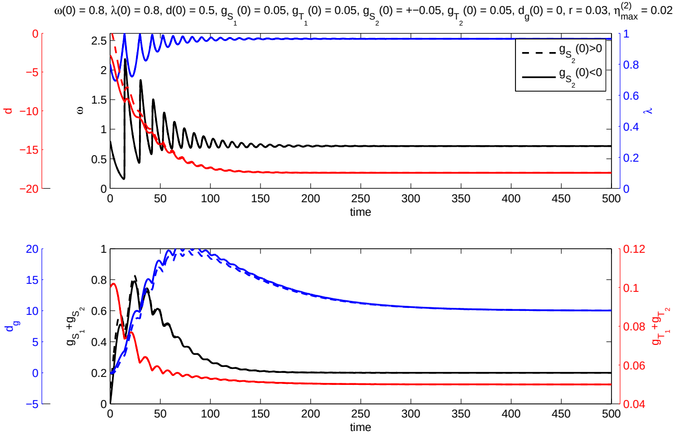
Example 5: Really bad initial conditions with timid government



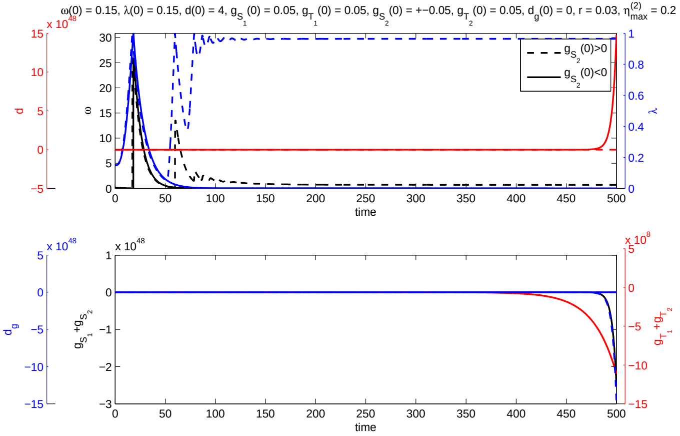
Example 6: Really bad initial conditions with responsive government



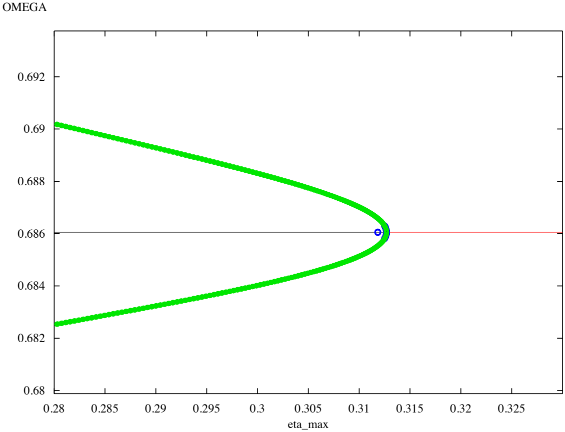
Example 7: Austerity in good times: harmless



Example 8: Austerity in bad times: a really bad idea



Hopft bifurcation with respect to government spending.



Next steps

- Introduce equities and household wealth effects in the consumption function (portfolio choices, etc)
- Extend to a stochastic model (stochastic interest rates, monetary policy, correlated market sectors, etc)
- Extend to an open economy model (exchange rates, capital flows, etc)
- Calibrate to macroeconomic time series

Jedi economics