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On the parameterization of Lindblad equations. (English summary)
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The celebrated Lindblad equation describes the time evolution of states (density operators) and observables (bounded selfadjoint operators) of open quantum systems (such as irreversible processes approaching equilibrium) in the same fashion in which the Schrödinger equation gives the unitary time evolution of closed quantum systems. It consists of a Hamiltonian (reversible) term and a dissipative term, given in terms of a set of (bounded) Lindblad operators V_J.

The purpose of this paper is to derive a parametrization of the Lindblad operators in terms of properties of the asymptotic state induced by them. For concreteness, the author restricts himself to the case of one Lindblad operator V for most of the discussion, but generalizes the result to the case of multiple V_J towards the end of the paper.

The central idea is to use the polar decomposition $V = U|V|$, where U is unitary (since V^*V is assumed to be invertible) and $|V| = (V^*V)^{1/2}$ is selfadjoint, and to define the operator $W = (V^*V)^{-1}$. One can then show that (under the assumption that $[W, H] = 0$) any initial state ρ_0 is asymptotically mapped into $P_W = W/\mathrm{tr}(W)$.

The author proceeds by introducing a "complete set of compatible observables" $\{Q_i, i = 0, 1, 2, \ldots\}$, consisting of mutually commuting operators, with $Q_0 = H$, the original Hamiltonian. The concluding step in the paper is to impose the KMS condition on the correlations of observables in the equilibrium state, which allows one (by analytic continuation) to rewrite the Lindblad equation in terms of the one-parameter automorphisms generated by the observables Q_i. This leads to the result that P_W is the grand canonical distribution associated with $\{Q_i\}$.

The paper relies heavily on technical results contained in two previous papers by the same author, but contains several gems, such as a critical discussion of the statement that the Lindblad equation is a "Markovian evolution equation" and the example provided to justify the introduction of more than one Lindblad operator, namely the disintegration of a molecule into some of its constituents according to QED.

Reviewed by *M. R. Grasselli*

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