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The quantum information manifold for ε-bounded forms. (English summary)

Dedicated to Professor Roman S. Ingarden on the occasion of his 80th birthday.

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The aim of the paper under review is to construct a Banach manifold inside the state space Σ of density operators (i.e., positive trace-class operators of trace one) on a Hilbert space from the viewpoint of quantum information geometry. Let $\mathcal{C}_p$, $0 < p < 1$, denote the space of compact operators A such that $|A|^p \in \mathcal{C}_1$, the trace-class operators. Let $\rho_0 \in \mathcal{C}_{\beta_0} \cap \Sigma$, $\beta_0 < 1$, and $H_0 = -\log \rho_0 + cI \geq I$ be a selfadjoint operator. By inductively applying a form-bounded perturbation technique started from ρ_0, the information manifold $\mathcal{M}(H_0)$ is constructed as a Banach manifold sitting inside $\bigcup_{0<p<1}(\mathcal{C}_p \cap \Sigma)$. The canonical affine structure, called the $(+1)$-affine structure, is introduced on $\mathcal{M}(H_0)$ together with the $(+1)$-affine connection. Furthermore, it is shown that the free energy $\Psi(\rho_X) := \log Z_X$ of the state $\rho_X = Z_X^{-1} e^{-(H_0+X)} \in \mathcal{M}(H_0)$ is infinitely Fréchet differentiable with a convergent Taylor series in a small neighborhood of ρ_X, from which it follows that the (-1)-coordinates (convex mixture coordinates) are analytic so that the manifold $\mathcal{M}(H_0)$ has a real analytic structure.

Reviewed by *Fumio Hiai*