A dynamical systems model for credit expansion, asset price bubbles and financial fragility

M. R. Grasselli

Introducion

Goodwin model

Keen model

Ponzi financing

Stabilizing government

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A dynamical systems model for credit expansion, asset price bubbles and financial fragility

M. R. Grasselli

Introduction

1. **Introduction**
   - Dynamic General Equilibrium views
   - Minskyian views

2. **Goodwin model**

3. **Keen model**

4. **Ponzi financing**

5. **Stabilizing government**
Dynamic General Equilibrium views

- Seeks to explain the aggregate economy using theories based on strong microeconomic foundations.
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- Collective decisions of rational individuals over a range of variables for both present and future.
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- The only way the economy can be in disequilibrium at any point in time is through decisions based on wrong information.
- Money is neutral in its effect on real variables.
- Largely ignores uncertainty by simply subtracting risk premia from all risky returns and treat them as risk-free.
Voices of discontent

- M. Morishima (1984): “If economists successfully devise a correct general equilibrium model (...) should it lack the institutional backing to realize an equilibrium solution, then [it] will amount to no more than a utopian state of affairs which bears no relation whatsoever to the real economy.”
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- K. Arrow (1986): “In the aggregate, the hypothesis of rational behavior has in general no implications.”

- R. Solow (2006): “Maybe there is in human nature a deep-seated perverse pleasure in adopting and defending a wholly counterintuitive doctrine that leaves the uninitiated peasant wondering what planet he or she is on.”
Minsky’s alternative interpretation of Keynes

- Neoclassical economics is based on barter paradigm: money is convenient to eliminate the double coincidence of wants.
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- Uncertainty in valuation of cash flows (assets) and credit risk (liabilities) drive fluctuations in real demand and investment.
- Economy is fundamentally cyclical, with each state (boom, crisis, deflation, stagnation, expansion and recovery) containing the elements leading to the next in an identifiable manner.
Minsky’s Financial Instability Hypothesis

- Start when the economy is doing well but firms and banks are conservative.
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- Ponzi financiers have to sell assets, liquidity dries out, asset market is flooded.
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- “Stability - or tranquility - in a world with a cyclical past and capitalist financial institutions is destabilizing.”
Goodwin Model (1967) - Assumptions

Assume that

\[ N(t) = N_0 e^{\beta t} \quad \text{(total labour force)} \]
\[ a(t) = a_0 e^{\alpha t} \quad \text{(productivity per worker)} \]
\[ Y(t) = \nu K(t) = a(t)L(t) \quad \text{(total yearly output)} \]

where \( K \) is the total stock of capital and \( L \) is the employed population.
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(total yearly output)

where \( K \) is the total stock of capital and \( L \) is the employed population.

- Assume further that

\[ \dot{w} = \Phi(\lambda)w \]  
(Phillips curve)

\[ \dot{K} = (Y - wL) - \delta K \]  
(Say’s Law)
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Goodwin Model - Differential equations

Define

\[ \omega = \frac{wL}{Y} = \frac{w}{a} \quad \text{(wage share)} \]

\[ \lambda = \frac{L}{N} = \frac{Y}{aN} \quad \text{(employment rate)} \]
Goodwin Model - Differential equations

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  \omega = \frac{wL}{Y} = \frac{w}{a} \quad \text{(wage share)}
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  \[
  \lambda = \frac{L}{N} = \frac{Y}{aN} \quad \text{(employment rate)}
  \]

- It then follows that

  \[
  \dot{\omega} = \omega (\Phi(\lambda) - \alpha) \quad (1)
  \]

  \[
  \dot{\lambda} = \lambda \left( \frac{1 - \omega}{\nu} - \alpha - \beta - \delta \right) \quad (2)
  \]
Goodwin Model - Properties

- If we take $\Phi$ to be linear, (1) reduces to the Lotka-Volterra equations for the predator-prey model.
Goodwin Model - Properties

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- To ensure \( \lambda \in (0, 1) \) we take

\[
\Phi(\lambda) = \frac{\phi_1}{(1 - \lambda)} - \phi_0
\]
Goodwin Model - Properties

- If we take $\Phi$ to be linear, (1) reduces to the Lotka-Volterra equations for the predator-prey model.
- To ensure $\lambda \in (0, 1)$ we take

$$\Phi(\lambda) = \frac{\phi_1}{(1 - \lambda)} - \phi_0$$

- Provided

$$\frac{1}{\nu} - \alpha - \beta - \delta > 0$$

$$\alpha + \phi_0 - \rho_1 > 0$$

(3)

are satisfied, the trivial equilibrium $(0, 0)$ is a saddle point.
Goodwin Model - Properties

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- To ensure $\lambda \in (0, 1)$ we take
  \[ \Phi(\lambda) = \frac{\phi_1}{(1 - \lambda)} - \phi_0 \]
- Provided
  \[ \frac{1}{\nu} - \alpha - \beta - \delta > 0 \]
  \[ \alpha + \phi_0 - \rho_1 > 0 \]
  are satisfied, the trivial equilibrium $(0, 0)$ is a saddle point.
- Moreover the only other equilibrium
  \[ (\bar{\omega}, \bar{\lambda}) = \left( 1 - \nu(\alpha + \beta + \delta), 1 - \sqrt{\frac{\phi_1}{\alpha + \phi_0}} \right) \]
  is non-hyperbolic.
Example 1: Goodwin model
Example 1 (continued): Goodwin model

\[ \omega_0 = 0.8, \lambda_0 = 0.9 \]
Goodwin Model - Extensions, structural instability, and empirical tests

- Desai 1972: Inflation leads to a stable equilibrium.
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- Ploeg 1985: CES production function leads to stable equilibrium.
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- Desai 1972: Inflation leads to a stable equilibrium.
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- Goodwin 1991: Pro-cyclical productivity growth leads to explosive oscillations.
- Solow 1990: US post-war data shows three sub-cycles with a “bare hint of a single large clockwise sweep” in the $(\omega, \lambda)$ plot.
- Harview 2000: Data from other OECD confirms the same qualitative features and shows unsatisfactory quantitative estimations.
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Testing Goodwin on OECD countries

Figure: Source: Harvie (2000)
Introducing a financial sector (Keen 1995)

- Assume now that new investment is given by

\[ \dot{K} = \kappa(1 - \omega - rd)Y - \delta K \]  \hspace{1cm} (5)

where \( \kappa(\cdot) \) is an increasing function the net profit share 
\( \pi = 1 - \omega - rd \).
Introducing a financial sector (Keen 1995)

- Assume now that new investment is given by
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- This leads to external financing through debt evolving according to
  \[ \dot{D} = \kappa(1 - \omega - rd)Y - (1 - \omega - rd)Y \]
Introducing a financial sector (Keen 1995)

- Assume now that new investment is given by
  \[
  \dot{K} = \kappa(1 - \omega - rd)Y - \delta K
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  where \( \kappa(\cdot) \) is an increasing function the net profit share \( \pi = 1 - \omega - rd \).
- This leads to external financing through debt evolving according to
  \[
  \dot{D} = \kappa(1 - \omega - rd)Y - (1 - \omega - rd)Y
  \]
- We take
  \[
  \kappa(x) = \kappa_0 + \kappa_1 e^{\kappa_2 x},
  \]
  for constants
  \[
  \kappa_0 < \nu(\alpha + \beta + \delta), \quad \kappa_1 > 0, \quad \kappa_2 > 0.
  \] (6)
Denote the debt ratio in the economy by $d = D/Y$, the model can now be described by the following system

\[
\begin{align*}
\dot{\omega} &= \omega \left[ \frac{\phi_1}{(1 - \lambda)^2} - (\alpha + \phi_0) \right] \\
\dot{\lambda} &= \lambda \left[ \frac{\kappa(1 - \omega - rd)}{\nu} - \alpha - \beta - \delta \right] \\
\dot{d} &= d \left[ r - \frac{\kappa(1 - \omega - rd)}{\nu} + \delta \right] + \kappa(1 - \omega - rd) - (1 - \omega)
\end{align*}
\]
Keen model - good equilibrium

- If we define

\[ \bar{\pi}_1 = \kappa^{-1}(\nu(\alpha + \beta + \delta)) = \frac{1}{\kappa_2} \log \left( \frac{\nu(\alpha + \beta + \delta) - \kappa_0}{\kappa_1} \right) \]

we see that one possible equilibrium for (7) is

\[ \bar{\omega}_1 = 1 - \bar{\pi}_1 - r \frac{\nu(\alpha + \beta + \delta) - \bar{\pi}_1}{\alpha + \beta} \]

\[ \bar{\lambda}_1 = 1 - \sqrt{\frac{\phi_1}{\alpha + \phi_0}} \]

\[ \bar{d}_1 = \frac{\nu(\alpha + \beta + \delta) - \bar{\pi}_1}{\alpha + \beta} \]
Keen model - Irrelevant equilibria

- Other equilibrium points are given by

\[(\bar{\omega}_2, \bar{\lambda}_2, \bar{d}_2) = (0, 0, \bar{d}_2)\]  \hspace{1cm} (9)

where \(\bar{d}_2\) is any solution of the equation

\[d \left[ r - \frac{\kappa(1 - rd)}{\nu} + \delta \right] + \kappa(1 - rd) - 1 = 0\]
Keen model - Irrelevant equilibria

- Other equilibrium points are given by

\[
(\omega_2, \lambda_2, \bar{d}_2) = (0, 0, \bar{d}_2)
\]  

where \(\bar{d}_2\) is any solution of the equation

\[
d \left[ r - \frac{\kappa(1 - rd)}{\nu} + \delta \right] + \kappa(1 - rd) - 1 = 0
\]

- Another set of equilibrium points are

\[
(\omega_3, \lambda_3, \bar{d}_3) = (0, \lambda, \bar{d}_1)
\]  

provided \(1 - r\bar{d}_1 = \bar{\pi}_1\), that is

\[
1 - r \frac{\nu(\alpha + \beta + \delta) - \kappa^{-1}(\nu(\alpha + \beta + \delta))}{\alpha + \beta} = \kappa^{-1}(\nu(\alpha + \beta + \delta))
\]
Example 2: convergent Keen model
Example 2 (continued): convergent Keen model

\( \omega_0 = 0.75, \lambda_0 = 0.75, d_0 = 0.1 \)
Keen model - Explosive debt

- If we rewrite the system with the change of variables $u = 1/d$, we obtain

\[
\dot{\omega} = \omega \left[ \frac{\phi_1}{(1 - \lambda)^2} - (\alpha + \phi_0) \right]
\]

\[
\dot{\lambda} = \lambda \left[ \frac{\kappa(1 - \omega - r/u)}{\nu} - \alpha - \beta - \delta \right]
\]

\[
\dot{u} = u \left[ \frac{\kappa(1 - \omega - r/u)}{\nu} - r - \delta \right] - u^2 \left[ \kappa(1 - \omega - r/u) - (1 - \omega) \right]
\]
Keen model - Explosive debt

- If we rewrite the system with the change of variables $u = 1/d$, we obtain

\[
\begin{align*}
\dot{\omega} &= \omega \left[ \frac{\phi_1}{(1 - \lambda)^2} - (\alpha + \phi_0) \right] \\
\dot{\lambda} &= \lambda \left[ \kappa \left( 1 - \omega - \frac{r}{u} \right) - \frac{\nu}{\nu} - \alpha - \beta - \delta \right] \\
\dot{u} &= u \left[ \kappa \left( 1 - \omega - \frac{r}{u} \right) - r - \delta \right] - u^2 \left[ \kappa \left( 1 - \omega - \frac{r}{u} \right) - (1 - \omega) \right]
\end{align*}
\]  

(11)

- We now see that $(0, 0, 0)$ is an equilibrium of (11) corresponding to the point

\[
(\bar{\omega}_4, \bar{\lambda}_4, \bar{d}_4) = (0, 0, +\infty)
\]  

(12)

for the original system.
Example 3: divergent Keen model
Example 3: divergent Keen model

\( \omega_0 = 0.75, \lambda_0 = 0.7, d_0 = 0.1 \)
Example 3 (continued): divergent Keen model

\[ \omega_0 = 0.75, \lambda_0 = 0.7, d_0 = 0.1 \]
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Data detour: debt

Private Debt as % of GDP

A graph showing private debt as a percentage of GDP over time, with data for different countries.
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Data detour: debt and employment

Figure: Source: Keen (2009)
Keen model - Local stability

- Analyzing the Jacobian of (7) and (11) we obtain the following conclusions.
Keen model - Local stability

- Analyzing the Jacobian of (7) and (11) we obtain the following conclusions.
- The good equilibrium \((\overline{\omega}_1, \overline{\lambda}_1, \overline{d}_1)\) is stable if and only if

\[
r \left[ \kappa'\left(\overline{\pi}_1\right) \left(\frac{\overline{\pi}_1}{\nu} - \delta\right) - (\alpha + \beta) \right] > 0.
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Keen model - Local stability

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  \[ r \left[ \kappa'(\bar{\pi}_1) \left( \frac{\bar{\pi}_1}{\nu} - \delta \right) - (\alpha + \beta) \right] > 0. \]
- The second equilibrium \((0, 0, \bar{d}_2)\) is unstable whenever
  \[ \kappa(1 - r\bar{d}_2) > \kappa(1 - \bar{\omega}_1 - r\bar{d}_1) \]
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Keen model - Local stability

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- The third equilibrium \((0, \lambda, \bar{d}_1)\) is structurally unstable.
Keen model - Local stability

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  \]
- The second equilibrium \((0, 0, \bar{d}_2)\) is unstable whenever
  \[
  \kappa(1 - r \bar{d}_2) > \kappa(1 - \bar{\omega}_1 - r \bar{d}_1)
  \]
- The third equilibrium \((0, \lambda, \bar{d}_1)\) is structurally unstable.
- The point \((0, 0, 0)\) is a stable equilibrium for (11) if and only if
  \[
  \frac{\kappa_0}{\nu} < \min(\alpha + \beta + \delta, r + \delta)
  \]
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Basin of convergence for Keen model
To introduce the destabilizing effect of purely speculative investment, Keen (2009) considers a modified version of the previous model with

$\dot{D} = \kappa(1 - \omega - rd)Y - (1 - \omega - rd)Y + P$

$\dot{P} = \Psi(g)Y$

where $\Psi(\cdot)$ is an increasing function of the growth rate of economic output

$g = \frac{\kappa(1 - \omega - rd)}{\nu} - \delta.$
Denoting \( \Psi(\omega, d) = \Psi\left(\frac{\kappa(1-\omega-rd)}{\nu} - \delta\right) \), the system now becomes

\[
\begin{align*}
\dot{\omega} &= \omega \left[ \frac{\phi_1}{(1 - \lambda)^2} - (\alpha + \phi_0) \right] \\
\dot{\lambda} &= \lambda \left[ \frac{\kappa(1 - \omega - rd)}{\nu} - \alpha - \beta - \delta \right] \\
\dot{d} &= d \left[ r - \frac{\kappa(1-\omega-rd)}{\nu} + \delta \right] + \kappa(1-\omega-rd) - (1-\omega) + p \\
\dot{p} &= \Psi(\omega, d) - \left( \frac{\kappa(1-\omega-rd)}{\nu} - \delta \right) p
\end{align*}
\]
Ponzi financing - Equilibria

- The finite debt equilibrium for the new system has $\pi_1$ and $\lambda_1$ as before, but

\[
\overline{\omega}_1 = 1 - \pi_1 - r\overline{d}_1
\]

\[
\overline{d}_1 = \frac{\nu(\alpha + \beta + \delta) - \pi_1 + \overline{p}_1}{\alpha + \beta}
\]

\[
\overline{p}_1 = \frac{\Psi(\alpha + \beta)}{\alpha + \beta}
\]

and is now unstable for typical model parameters.
The finite debt equilibrium for the new system has $\bar{\pi}_1$ and $\bar{\lambda}_1$ as before, but

$$\bar{\omega}_1 = 1 - \bar{\pi}_1 - r\bar{d}_1$$

$$\bar{d}_1 = \frac{\nu(\alpha + \beta + \delta) - \bar{\pi}_1 + \bar{p}_1}{\alpha + \beta}$$

$$\bar{p}_1 = \frac{\Psi(\alpha + \beta)}{\alpha + \beta}$$

and is now unstable for typical model parameters.

On the other hand, introducing $u = 1/d$ and $\nu = 1/p$ we find that

$$(\omega, \lambda, d, p) = (0, 0, +\infty, -\infty)$$

$$(\omega, \lambda, d, p) = (0, 0, +\infty, +\infty)$$

correspond to stable equilibria for the modified system.
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Example 4: effect of Ponzi financing
Introducing a government sector

- A final extension proposed by Keen (echoing Minsky) consists of adding government spending and taxation into the original system according to

\[
\dot{G} = \Gamma(\lambda)Y \\
\dot{T} = \Theta(\pi)Y
\]
Introducing a government sector

A final extension proposed by Keen (echoing Minsky) consists of adding government spending and taxation into the original system according to

\[ \dot{G} = \Gamma(\lambda)Y \]
\[ \dot{T} = \Theta(\pi)Y \]

Defining \( g = G/Y \) and \( t = T/Y \), the net profit share is now

\[ \pi = 1 - \omega - rd + g - t \]
Introducing a government sector

- A final extension proposed by Keen (echoing Minsky) consists of adding government spending and taxation into the original system according to

\[ \dot{G} = \Gamma(\lambda)Y \]
\[ \dot{T} = \Theta(\pi)Y \]

- Defining \( g = G/Y \) and \( t = T/Y \), the net profit share is now

\[ \pi = 1 - \omega - rd + g - t \]

- The new 5-dimensional system displays more local fluctuations, but no breakdown for the same initial conditions as before.
Example 5: stabilizing government

\[ w_0 = 0.75, \lambda_0 = 0.7, d_0 = 0.1, g_0 = -0.14, t_0 = 0.39, r = 0.03 \]
Example 5 (continued): stabilizing government

\[ w_0 = 0.75, \lambda_0 = 0.7, d_0 = 0.1, g_0 = -0.14, t_0 = 0.39, r = 0.03 \]
Next steps

- Model prices for capital goods $P_k$ and commodities $P_c$ explicitly (Kaleckian mark-up theory, inflation, etc)
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- Calibrate to macroeconomic time series.
Concluding thoughts

- Solow (1990): The true test of a simple model is whether it helps us to make sense of the world. Marx was, of course, dead wrong about this. We have changed the world in all sorts of ways, with mixed results; the point is to interpret it.
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- Schumpeter (1939): Cycles are not, like tonsils, separable things that might be treated by themselves, but are, like the beat of the heart, of the essence of the organism that displays them.