The Heroic Age: the intelectual enterprise of Greek mathematicians in the V century BC

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26/10/1995

1 The Epoch

Somewhere between the probable date for Pythagora's death and the foundation of Plato's Academy lies the fifth century BC, an era which witnessed an unsual intelectual ebullience in Greece, particularly in Athens, from where we all know the artistic, literary and political achievements of the Periclean century.

The mathematics from this period, however, is comparatively less studied and popularized nowadays, when everyone knows who was Thales or Archimedes and Euclid, but few discuss the mathematical achievements of those contemporary to Socrates.

Nevertheless, the depth of the problems that were treated, the mathematical meaning of the changes that were introduced and the philosophical scope of the methods and questions tackled, particularly if combined with the lack of scientific resources of the time, allow us to justifiably call by The Heroic Age the period that we will briefly analise in this paper through the work of some of its mathematicians.

2 The Mathematicians

Coming form Ionia, having been influenced by the mathematics and philosophy of Tales, the philosopher of Nature Anaxagoras of Clazomenae settled down in Athens. From his life, we mainly know that he was arrested when his intelectual inquires led him to claim that the Sun was not divine and that the Moon must have had inhabitants. It is interesting to speculate exactly which one of these statements was the most serious threat to the accepted wisdon of the time. We suggest that the existence of life beyond Earth, the possible centre of the Universe, makes its importance only relative, therefore generating a permanent source of controversy , which would resurface again in the work of Plutarco and much later in Kant. The absence of divinity in the Sun, on the other hand, had already been accepted by civilizations much older than the Greece and does not play an important role even in the elaborate Greek mithology. The arrest also had some mathematical importance, since it was during his time in prison that Anaxagoras proposed for the first time the classical problem of the quadrature of a circle, introducing there and then a clear distinction between pragmatic interest and purely intelectual research. Although not having explicitly stated the precise conditions for the quadrature problem, it is clear that Anaxagoras was seeking an exact result, hardly of any practical applicability.

The same region, the Dodecanese islands, was the origin of Hippocrates of Chios, who was heading to Athens when he was the victim of a misfortune, perhaps a fraud, perhaps the attack of pirates, which left him with no money and forced him into working as a geometer, instead of his original intention of working as a merchant. It is due to him the statement, and less likely the proof, of the oldest theorem about curved figures in the Ancient World: similar portions of a circle are proportional to the square of their bases. He was then naturally inclined to attempt the squaring of a circle, and along the way, using his theorem, he succeded in squaring three types of lunes, where we observe a grasping of geometrical entities which had been absent in the mathematical activities of the day. In trying to square rectangles using proportions, he found what is known as extended proportion, where one observes the appearance of the cubic root of a given number, putting him in slight contact with the problem of duplicating a cube, another of the classical problems proposed in this era, whose legendary origin (the duplication of the altar of the Delian Oracle) is well known.

The last of the three classical problems of Greek geometry also appeared and circulated in Athens during this heroic century, being mainly treated by the sophist Hippias of Elis. It concerns the trisection of an angle, solved by Hippias usign a curved discovered by him, which later received the name of quadratrix of Hippias, since it can also be used to square the circle. It is not known, however, if this latter use was proved by its author, since it involves a limiting procedure which was beyond the mathematical capabilities of the time.

The mathematician who worked directly in the problem of duplicating the cube was Arquitas, the ruler of Tarentum and a friend of Plato's, who was a disciple of Filolaus, therefore being a remanescent Pythagorean. His contribution to music (the harmonic mean, research on the relation between tones and air movement) are as well known as his introduction of the mathematical quadrivium - arithmetics, music, geometry and astronomy - into the realm of liberal disciplines. His solution of the Delian problem by means of the intersection of three curved surfaces, without the help of analytic geometry, shows us how far the sophistication of a mathematical endevour can get when all the efforts are concentrated in the ellucidation of a problem.

Apart from the classical problems, the mathematical community of the time was stirred by the discovery of the incommensurables, allegedly due to the Pythagorean Hippasus of Metapontum, who is likely to have been expelled from the creed for revealing his findings, which deeply challenged the philosofical properties of numbers held by the Pythagoreans.

Another philosophical crisis was originated by Zeno of Elea, a disciple of Parmenides. His four most important paradoxes intended to contradic the hypotheses that space and time were infinitely divisible (as in the Dicothomy and the Achilles and the Tortoise paradoxes) and at the same time to contradict its negation, that is, that space and time have a fudamental unit which cannot be further divided (the arrow and the stadium paradoxes). The implications of such paradoxes ultimately led modern mathematicians to the theory of limits and the topological concepts of continuity and density. Their format reappeared several times in modern philosophy, apart from having influenced the Socratic method for investigation of the truth.

To complete our picture of fifth century BC Greek mathematicians, let us mention Democritus, well known for his chemical atomism, but equally as important for his geometrical atomism, through the use of infinitesimals in the argument for finding areas and volumes. The infinitesimals are intimately connected with the incommensurables and with Zeno's paradoxes, and their use in mathematical analysis was only formalized a couple of decades ago.

3 Conclusion

Those were, in general terms, the mathematicians and themes of the Heroic Age. To judge the mathematical activities of the V century BC by saying that the classical problems are impossible to solve with ruler and compass, that half of the paradoxes had no solid basis or that the infinitesimals were not considered rigorous until much later, is to poorly realize by what extent the crisis, errors and questionings are important for the development of mathematics and science in general.

Classical problems, paradoxes of motion and infinitesimals formed the ambitious basis for mathematical investigation upon which this science developed in Greece, leading to research and excitment that crossed centuries to the Western world until periods very close to our own time.

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