In search of the Minsky moment

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Outline

Minsky's Financial Instability Hypothesis

- Start when the economy is doing well but firms and banks are conservative (perhaps because of memory of previous crisis).
- Most projects succeed - "Existing debt is easily validated and units that are heavily in debt prospered: it pays to lever".
- Revised valuation of cash flows, exponential growth in credit, investment and asset prices.
- Highly liquid, low-yielding financial instruments are devalued, rise in corresponding interest rate.
- Beginning of "euphoric economy": increased debt to equity ratios, development of Ponzi financier.
- Viability of business activity is eventually compromised.
- Ponzi financiers have to sell assets, liquidity dries out, asset market is flooded.
- Euphoria becomes a panic.

Defintion

- Consider a representative agent solving

$$\sup_{c} E_t \left[\sum_{j=1}^{\infty} \beta^{j-t} u(c_j) \right]$$

for exogenously given (e_t, d_t).

- Denoting $q_t = u'(e_t + d_t)p_t$, the FOC for optimality give

$$q_t - \beta E_t [q_{t+1}] = \beta E_t \left[d_{t+1} u'(e_{t+1} + d_{t+1}) \right]$$

- The general solution is of the form $q_t = F_t + B_t$ where

$$F_t = \sum_{j=1}^{\infty} \beta^j E_t \left[d_{t+j} u'(e_{t+j} + d_{t+j}) \right]$$

is the fundamental price and B_t is a bubble term satisfying

$$E_t[B_{t+1}] = \beta^{-1} B_t \qquad (1)$$

Positivity and inception

- The general form for B_t satisfying (1) is

$$B_t = \beta^{-t} B_0 + \sum_{s=1}^{t} \beta^{s-t} z_s, \quad E_t[z_{t+1}] = 0. \tag{2}$$

- Observe that it follows directly from (1) that

$$E_t[B_{t+j}] = \beta^{-j} B_t, \quad \forall j > 0. \tag{3}$$

- Since $\beta^{-1} > 1$, we see that $E_t[q_{t+j}] \to \pm\infty$.
- Given free disposal, we conclude that $B_t \geq 0$ for all t.
- But this implies that $z_{t+1} \geq -\beta^{-1} B_t$ for all t.
- Now if $B_s = 0$ for some s, then $z_{s+1} \geq 0$.
- But since $E_s[z_{s+1}] = 0$ we see that $z_{s+1} = 0$ a.s.
- Therefor any nonzero rational bubble must start with $B_0 > 0$.

Rational expectations

Consider a model with finitely many infinitely lived agents with diverse information and rational expectations.

Proposition (Tirole, 1982)

1. *In a stock market with horizon $T < \infty$, bubbles are all equal to zero for all agents.*
2. *In the infinite horizon case, bubbles satisfy*

$$B(s_t^i, p_t) = \beta^T E[B(s_{t+T}^i, p_{t+T}) | s_t^i, S_t(p_t)].$$

3. *Whether short-sales are allowed or not, bubbles do not exist in a fully dynamic REE and*

$$F(s_t^i, S_t(p_t)) = p_t.$$

Overlapping generations

- An alternative is to consider overlapping agents in a Diamond (1965) growth model.
- This consists of consumers who live for two periods and have utility $u(c^y, c^o)$
- Define wages w_t, production function $Y_t = L_t f(k_t)$ (for labor force L_t and capital stock k_t), savings function $s(w_t, r_{t+1})$, and real interest rate $r_t = f'(k_t)$.
- These assumptions uniquely define an asymptotic real interest rate $\bar{r}$.
- Tirole (1985) then shows that a bubble can exist provided $0 < \bar{r} < g$, where g is the rate of growth of the economy.

The Efficient Markets Hypothesis

- Denote $R_{t+1} = \frac{p_{t+1} - p_t + d_{t+1}}{p_{t+1}}$.
- As we have seen, a first-order rational expectations condition for risk-neutral agents lead to

$$E_t[R_{t+1}] = 1 + r. \quad (4)$$

- Solving this recursively leads to

$$p_t = \sum_{j=1}^{\infty} \frac{1}{(1+r)^j} E_t[d_{t+j}], \quad (5)$$

  plus a possible rational bubble term satisfying $E_t[B_{t+1}] = (1 + r)B_t$.
- Either (4) or (5) can be taken as an EMH.
- Statistical tests on actual returns indicate that they are not *very* forecastable, leading to the conclusion that the EMH cannot be rejected.

Volatility bounds

- Suppose that $p_t = E_t[p_t^*]$, where p_t^* is a perfect foresight price.
- Then $p_t^* = p_t + \varepsilon_t$, where ε_t is the forecast error and is uncorrelated with p_t.
- It follows that $\sigma(p_t) \leq \sigma(p_t^*)$.
- This, however, is found to be dramatically violated by data (Shiller (1981)).

Violation of Volatility Bounds

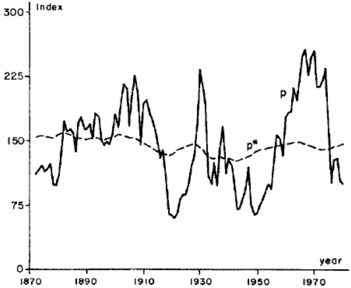


FIGURE 1

Note: Real Standard and Poor's Composite Stock Price Index (solid line p) and *ex post* rational price (dotted line p^*), 1871–1979, both detrended by dividing a long-run exponential growth factor. The variable p^* is the present value of actual subsequent real detrended dividends, subject to an assumption about the present value in 1979 of dividends thereafter. Data are from Data Set 1, Appendix.

FIGURE 2

Note: Real modified Dow Jones Industrial Average (solid line p) and *ex post* rational price (dotted line p^*), 1928-1979, both detrended by dividing by a long-run exponential growth factor. The variable p^* is the present value of actual subsequent real detrended dividends, subject to an assumption about the present value in 1979 of dividends thereafter Data are from Data Set 2, Appendix.

Figure: Source: Shiller (1981)

Alternative models (Shiller, 1984)

- Consider a model where sophisticated investors have a demand function (portion of shares) of the form

$$Q_t^i = \frac{E_t[R_{t+1}] - \alpha}{\phi}. \tag{6}$$

- In addition, suppose there are noise traders who react to fads Y_t through a demand function $Q_t^n = Y_t/p_t$.
- In equilibrium we have $Q_t + \frac{Y_t}{p_t} = 1$.
- Inserting this into (6) and solving recursively leads to

$$p_t = \sum_{j=1}^{\infty} \frac{E_t[d_{t+j}] + \phi E_t[Y_{t-1+j}]}{(1+\alpha+\phi)^j}. \tag{7}$$

- This is also consistent with prices being not very forecastable.

Noise Trader Risk (DeLong, Shleifer, Summers and Waldmann, 1990)

- Consider a safe asset (s) with perfectly elastic supply paying a dividend leading to a constant price 1 and an unsafe asset (u) with fixed unit supply and the same dividend rate.
- Suppose that a proportion μ of the agents are noise traders.
- According to their beliefs when young, all agents want to maximize the expected values of an identical utility $u(w) = -e^{-2\gamma w}$, where w is their wealth when old.
- Sophisticated investors accurately perceive the distribution of (u), whereas noise traders young at t misperceives its expected value by an i.i.d random variable

$$\rho_t \sim N(\rho^*, \sigma_\rho^2)$$

Equilibrium price

- After each group maximizes their utility, at equilibrium we have $(1-\mu)Q_t^i + \mu Q_t^n = 1$.
- This leads to the pricing equation

$$p_t = \frac{1}{1+r}\big(r + E_t[p_{t+1}] + \mu\rho_t - 2\gamma \mathrm{Var}_t[p_{t+1}]\big).$$

- Assuming stationary unconditional distributions, we find the steady state solution

$$p_t = \underbrace{1}_{\text{fundamental}} + \overbrace{\frac{\mu(\rho_t - \rho*)}{1+r}}^{\text{misconceptions at } t} + \underbrace{\frac{\mu\rho^*}{r}}_{\text{price pressure}} - \overbrace{\frac{2\gamma\mu^2\sigma_\rho^2}{(1+r)^2}}^{\text{compensation}}$$

Financial Intermediation (Allen and Gale, 2000)

- Suppose there is a continuum of small, risk-neutral investors with no wealth of their own and a continuum of small, risk-neutral banks with $B > 0$ funds to lend at rate r trading at $t = 1, 2$.
- Consider a safe asset (s) with return $(1 + r)$ and a risky asset (R) with price at $t = 2$ given by a random variable p_2 with density $h(p_2)$ on $[0, p_2^{\max}]$ and mean $\overline{p_2}$.
- In addition, there is a production function $f(x)$ for the economy and an investment cost $c(x)$.

Existence of bubbles

- A representative investor needs to choose quantities Q_1^s and Q_1^R of the safe and unsafe assets at time $t = 1$ at prices 1 and p_1, respectively.
- The equilibrium price in the presence of banks is then

$$p_1 = \frac{1}{1+r}\left[\frac{\int_{(1+r)p_1}^{p_2^{\max}} p_2 h(p_2)dp_2 - c'(1)}{\mathrm{Prob}[p_2 \geq (1+r)p_1]}\right]. \qquad (8)$$

- Define the fundamental value as the price that an investor would pay if he had to use his own money $B > 0$.
- This leads to

$$p_1^F = \frac{\overline{p_2} - c'(1)}{1+r}. \qquad (9)$$

- We can then show that $p_1 \geq p_1^F$ with strict inequality iff $\mathrm{Prob}[p_2 < (1+r)p_1] > 0$

Liquidity preferences

- An asset is illiquid if its liquidation value at an earlier time is less than the present value of its future payoff.
- For example, an asset can pay $1 \leq r_1 \leq r_2$ at dates $T = 0, 1, 2$.
- Let $(r_1 = 1, r_2 = R)$ be an illiquid asset and $(r_1 > 1, r_2 < R)$ be a liquid one.
- At time $t = 0$, consumers don't know in which future date they will consume.
- The consumer's expected utility is

$$pU(r_1) + (1 - p)U(r_2),$$

  where p is the proportion of early consumers.
- Sufficiently risk-averse consumers prefer the liquid asset.
- A similar story holds for entrepreneurs.

A model for a bank, Diamond and Dybvig (1983)

- Banks borrow short and lend long.
- Suppose a bank offers a liquid asset ($r_1 = 1.28, r_2 = 1.813$) to 100 depositors each with \$1 at $t = 0$.
- In addition, the bank can invest in an illiquid asset ($r_1 = 1, r_2 = 2$).
- If $w = 1/4$, the bank needs to pay $25 \times 1.28 = 32$ at $t = 1$.
- At $t = 2$ the remaining depositors receive $\frac{68 \times 2}{75} = 1.813$ and the bank is solvent.
- This is a Nash equilibrium is *all* depositors expect only 25 to withdraw at $t = 1$.
- *But* liquidity preferences are unverifiable private information.
- Another Nash equilibirum consisting of *all* depositors forecasting that everyone will withdraw at $t = 1$.

A model for interbank loans, Allen and Gale (2000)

- Consider a Diamond and Dybvig model with a liquid asset $(1, 1)$ and an illiquid asset $(r < 1, R > 1)$.
- Consumer preferences are given by,

$$U(c_1, c_2) = \begin{cases} u(c_1) & \text{with probability } w \\ u(c_2) & \text{with probability } (1-w) \end{cases}$$

- The economy is divided into 4 identical regions labeled A, B, C, D, each corresponding to a single bank (or a representitive bank).
- The probability w, varies from region to another and can take one of two values, w_H and w_L.

Table: Regional Liquidity Shocks

	A	B	C	D
S_1	w_H	w_L	w_H	w_L
S_2	w_L	w_H	w_L	w_H

Optimal interbank loans - centralized solution

- Banks can invest in either the liquid or illiquid assets and promise consumption (c_1, c_2) to consumers.
- The **centralized solution** consists of the best allocation at time $t = 0$

$$\begin{aligned} \gamma c_1 &= y \\ (1-\gamma)c_2 &= Rx \end{aligned}$$

where $\gamma = \dfrac{w_H + w_L}{2}$ is the fraction of early consumers.

- At time $t = 1$ the planner transfer the $(\gamma - w_L)c_1 = (w_H - \gamma)c_2$ excess resources from two of the regions to the other two.

Optimal interbank loans - decentralized solution

- Banks exchange deposits at $t = 0$, each of a total amount $z_i = (w_H - \gamma)$, and when faced with liquidity shortange they follow the 'pecking order'.

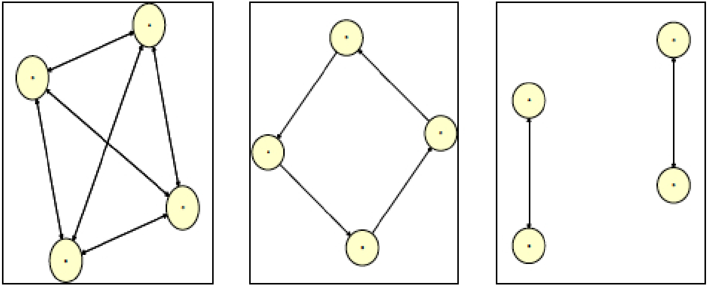


Figure: Networks, complete connected (left), incomplete connected (middle), incomplete disconnected (right)

Shocks and stability

- We then consider external shocks of the forms

Table: Regional Liquidity Shocks

	A	B	C	D
S_1	w_H	w_L	w_H	w_L
S_2	w_L	w_H	w_L	w_H
$\bar{S}$	$\gamma + \varepsilon$	γ	γ	γ

- Allen and Gale then prove that in the incompletely connected case, if bank A went bankrupt, and accordingly causing bank D to bankrupt, then all other banks must go also bankrupt at $t = 1$.
- More importantly, for the same parameter values that caused bank A in the previous case to default, there exist we can find an equilibrium with completely connected networks that does not involve runs in state $\bar{S}$.

Our model, The summarized story

- Society
- Liquidity Preference
- Searching for partners
- Learning and Predicting
- Bank birth
- Interbank Links
- Contagion

Society

- We have a society of individuals investing at the beginning of each period.
- There is a shock to their preferences at the mid of the period
- If the shock is big enough the individual would have wished he made his investment differently.
- For each individual i, an initial preference is drawn from a continuous uniform random variable U_i
- If $U_i < 0.5$ the investor is set to be liquid asset investor, otherwise he is long term asset investor.
- At time $t = 2$, $W_i = \mid U_i + (-1)^{ran_i} \frac{\epsilon_i}{2} \mid$
- If $W_i < 0.5$ the investor wants to be a short term investor, otherwise he wants to be long term investor
- Because of anticipated shocks, individuals explore the society searching to partners to exchange investments.

Searching for partners

- We impose some constrains on the individual capacity to go around and seek other individuals to trade.
- This reflects the inherited limited capability of information gathering and environment knowledge of individual agents.
- We use a combination of Von Neumann and Moore neighborhood:

5	1	6
2	X	3
7	4	8

To join or not to join a bank

- Assume a bank offers a fixed contract promising a payment of $c_1 > 1$ at $t = 1$ for each unit (dollar) deposited and $1 < c_2 < R$ for $t = 2$ under the assumption there is no bank run.
- The an agent will join if:
  1. has short term preferences and expects NOT to change preference for the coming day
  2. has short term preferences, expects to change preference and NOT find a partner to trade
  3. has long term preferences, expects to change preference
- The agent will NOT join if:
  1. has short term preferences, expects to change and believes he can find a partner
  2. has long term preferences and is confident they will not change

Bank birth

- We follow the work of Following Howitt and Clower (1999,2007) on the emergence of economic organizations
- With probability $0 < h < 1$ an agent will have the 'idea of entrepreneurship'
- Market search for an opportunnity to establish a bank
- Establish a bank if he can find x and y such that $x + y \leq 1$ and

$$y = c_1 W_i$$
$$Rx = c_2(1 - W_i)$$

- Individuals become aware of bank existence only if the bank lies in their neighborhood
- In addition we give the bank the reach of its new members

Experiments: In a perfect bank world

- Probability of being hit with bank idea $h = 0.9$.
- 50 time steps
- Promised payoff $c_1 = 1.1$, $c_2 = 1.5$ and $R = 2$.

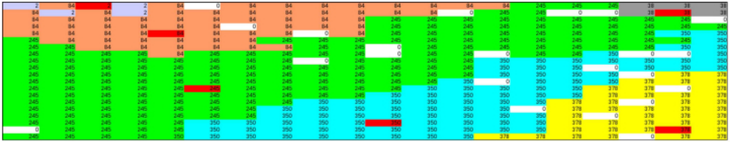


Figure: Banks established.Banks highlighted in red, while other colors indicating individuals joined banks [245 350 84 378 2 38]

Next steps

- Need to incorporate bank run
- Individuals moving between banks
- Banks form a new kind of agents that can in turn trade with each other (form links), and form their strategies to predict the number of early customers.

Goodwin's Model

- Let $N = n_0 e^{\beta t}$ be the labour force, $a = a_0 e^{\alpha t}$ be its productivity and $\lambda = L/N$ be the employment rate.
- Define the total output $Y = aL$ and total capital as $K = \nu Y$.
- Assume that wages satisfy

$$\frac{dw}{dt} = F_w(\lambda) w,$$

where $F_w(\lambda)$ is a Phillips curve.

- Let the wages share of total output be ω and profit share be $\pi = 1 - \omega$.
- Suppose further that the rate of new investment is given by

$$I = \frac{dK}{dt} = (1 - \omega) Y - \gamma K$$

Differential Equations

- It is easy to deduce that this leads to

$$\frac{d\omega}{dt} = \omega(F_w(\lambda) - \alpha) \tag{10}$$

$$\frac{d\lambda}{dt} = \lambda\left(\frac{1-\omega}{\nu} - \alpha - \gamma - \beta\right) \tag{11}$$

- This system is globally stable and leads to endogenous cycles of employment.

Keen's extended model

- Consider the same model as before, but with a Phillips-type investment function $I_g = k(\pi_n)$ of the net profit share is
$$\pi_n = 1 - \omega - rd,$$
where $d = D/Y$ and the absolute debt level D evolves according to
$$\frac{dD}{dt} = I_g - \pi_n = rD + k(\pi_n) - (1 - \omega)$$

Differential Equations

- The corresponding dynamical systems now reads

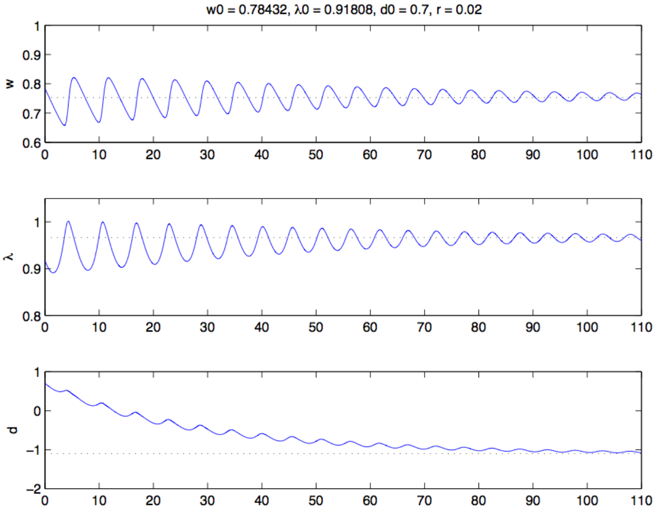
$$\frac{d\omega}{dt} = \omega(F_w(\lambda) - \alpha) \tag{12}$$

$$\frac{d\lambda}{dt} = \lambda\left(\frac{k(\pi_n)}{\nu} - \alpha - \gamma - \beta\right) \tag{13}$$

$$\frac{dd}{dt} = k(\pi_n) - (1 - \omega) - d\left(\frac{k(\pi_n)}{\nu} - \gamma\right) \tag{14}$$

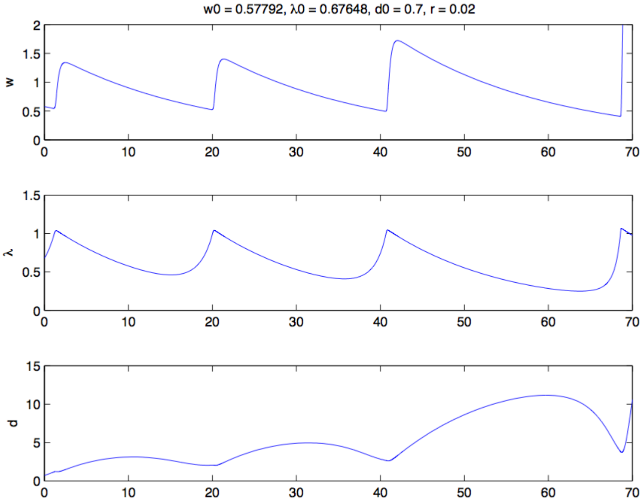
- This system is locally stable but globally unstable.

Example 1: convergence to equilibrium



Example 1: convergence to equilibrium (continued)

Example 2: financial meltdown



Example 2: financial meltdown (continued)

Next steps

- Add government (regulatory) sector.
- Incorporate asset prices explicitly.
- Introduce noise (stochastic interest rates, risk premium, etc)
- Move to systems of SDE
- Thanks !