McMaster University Arts and Science Winter 2013
Final Exam — PRACTICE version
Duration: 3 hours

Instructor: Dr. D. Haskell

Name: ____

Student ID Number: _______________

This test paper is printed on both sides of the page. There are 12 question on 12 pages. You are responsible for ensuring that your copy of this test is complete. Bring any discrepancies to the attention of the invigilator. More paper for rough work is available from the invigilator.

Instructions

(1) Only the standard McMaster calculator is allowed.
(2) All answers must be written in the space following the question. If you write an answer on scratch paper, indicate **clearly** where to find your answer.

This PRACTICE version of the midterm is intended to give you an idea of the format, approximate length and approximate difficulty of the actual midterm. There is no guarantee as to the actual length and difficulty of the actual exam. In particular, the actual midterm will NOT be "just the same with the numbers changed".

1) [10 points]

a) State the Integral Test for convergence of the series $\sum_{n=0}^{\infty} a_n$.

b) State the Fundamental Theorem of Calculus Part I.

c) Find $\int_1^2 x^2 \, dx$.

d) The volume V of a right circular cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. Find the rate of change of V as r changes and as h changes.

e) Sketch the slope field for the differential equation $\frac{dy}{dx} = y$.

2) [10 points]

a) Sketch a graph of a function $y = f(x)$ and show how, starting from some value x_1, Newton's method is used to find an approximation x_2 to the solution of the equation $f(x) = 0$.

b) Give an example of a sequence which is bounded but not monotonic.

c) Is the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ convergent or divergent. Justify your answer briefly.

d) Suppose $x(t)$, $y(t)$ are two interacting populations described by the differential equations

$$\frac{dx}{dt} = 0.01x - 0.5xy \qquad \frac{dy}{dt} = -0.01y + 0.002xy \ .$$

Which variable represents the predator population, and which is the prey? Explain briefly.

e) Find $\sum_{n=1}^{100} 2$.

3) [8 points]

a) State the domain of f if $f(x,y) = \dfrac{\sqrt{4-x^2}}{y^2+3}$.

b) Sketch the level curve $k = 4$ for $f(x,y) = \sqrt{x^2+y^2}$.

c) Does the following limit exist? Justify your answer.

$$\lim_{(x,y)\to(0,0)} \frac{-xy}{x^2+y^2}$$

4) [8 points]

a) Use the fourth degree Taylor Polynomial of $\cos(2x)$ to find $\lim_{x \to 0} \frac{1-\cos(2x)}{3x^2}$.

b) Find the radius and interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{5^n(n+3)}(x-2)^n.$$

5) [8 points] Find the indefinite integral $\displaystyle\int \frac{3x-1}{x^2-3x-10}\,dx$.

6) [8 points] Find the critical points of the function $f(x) = x^{2/3}(6-x)^{1/3}$. Classify them as local maxima or minima using the first or second derivative test. Decide if there are any absolute maxima or minima.

7) [8 points] Test the series $\sum_{n=2}^{\infty} \frac{8^n}{2+9^n}$ for convergence. State any test you use, and be careful to check its hypotheses.

9) [8 points] Solve the differential equation: $x\ln(x)\dfrac{dy}{dx} + y = xe^x$.

10) [8 points] Find the improper integral $\displaystyle\int_1^\infty (1 - \frac{1}{x})e^{\ln(x) - x}\ dx$.

11) [8 points] Decide if the following series converge absolutely.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n+3}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)}{n^3}$

12) [12 points]

a) What can you say about solutions to the differential equation $\frac{dy}{dx} = x^2 + y^2$ just by looking at the differential equation?

b) Sketch the graph of a function that illustrates how the hypotheses are needed in the Intermediate Value Theorem.

c) Suppose $\sum a_n$ is a convergent sum of positive terms. Does it follow that $\sum (-1)^n a_n$ converges?