ARTWORK BY CHRISTIE CHAN

EULER'S SEVEN BRIDGES

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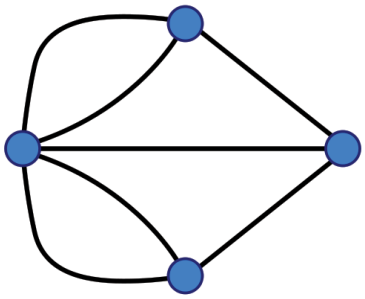
How Island Hopping Changed Math Forever

During stressful times, it can be beneficial to turn our thoughts to the goings-on of 18th century Prussian schoolchildren. In particular, let's take a look at a game that was popular among those who lived on the banks of the Pregel River, in a city then-called Konigsberg (now known as Kaliningrad). The river splits the city in two, essentially creating a pair of islands smack-dab in the middle of the downtown core. A system of seven bridges connects each of the disconnected pieces of land, as you can see in the picture above.

The game goes like this: start on one of the land masses and see if you can cross every bridge without ever retracing your path. It doesn't matter where you start or where you finish, as long as you never touch the same spot twice. Fun, right?! If you don't believe that it's an absolute blast, give it a try! This is potentially the only opportunity you'll ever have to test out your Prussian schoolchild impersonation judgement-free. See if you can trace out a path with your finger that crosses each bridge once and only once. If you're feeling ambitious, draw the picture on a separate piece of paper and go at it with a pencil. If you're feeling ridiculous, rearrange your room, putting your bed in the middle and painting the floors blue to resemble the Pregel River. Pinterest will love it.

After enough experimentation, you'll probably realize that the task is impossible. You'll always end up doubling up on a bridge at some point in order to cross them all. But *why?* This is the question prolific Swiss mathematician and all-time badass Leonhard Euler asked himself in 1735 after hearing about the popular game. After staring at a map and examining the bridge system for long enough, Euler came to a revelation that proved pivotal in the development of modern mathematics.

Euler's ingenious insight was that the picture we've been using to solve the problem puts too much emphasis on the land masses and doesn't adequately represent the connections between them. In other words, it doesn't actually matter how we draw the land masses; all that matters is the configuration of the bridges. With this in mind, Euler redrew the picture in the following way:



Before I keep going, you should take a few seconds to convince yourself that this new picture is identical to the one we were looking at before. Each dot represents a piece of land, and the lines represent the bridges between them. The creative minds who determine mathematical nomenclature decided to call this type of picture a *graph*. Now prepare yo'self, because we about to get real mathy up in hurr.

Looking at the graph, you will see that each blue dot is touched by an odd number of bridges – one by 5 bridges and the rest by 3. Herein lies the problem! It turns out that if a land mass has connections to an odd number of bridges, it must be either the start or the end of our journey. Why is that? Well, the land masses in the middle of the path must have an even number of bridges, half for 'arrival' and half for 'departures'. The start and finish, on the other hand, can have an odd or even number of bridges (try it out!).

So, according to this logic, every land mass in this situation must either be the start or the finish of our path. But we've got a total of four spots to visit, so this is clearly impossible! *Boom!* That right there, friends, was a mathematical proof. We did it – no, no – *you* did it.

Well, actually, Euler did it, and his clever trick of turning the Seven Bridges of Konigsberg into a graph led to the birth of a new branch of math, appropriately called *graph theory*. It's become a huge area of study and plays a role in analyzing – among other things – biological systems, social and economic networks, linguistic structures, and the internet. Wanna know how Google's page rank works? Graph theory. Curious why detectives' walls always have pictures of suspects' faces connected with yarn? Graph theory. Ever wonder how every single world map can be coloured using only four colours? Uh, yeah, I'm gonna go with graph theory.

Most people credit Euler with the foundation of graph theory, and this acclamation is certainly justified. I believe, however, that some praise belongs to the schoolyard children of the world, whose island-hopping whimsies helped create an immensely powerful mathematical tool. And who knows? Maybe your favourite jungle-gym game or driveway divertissement holds the key to math's next big leap. ■