Arts & Science 1D06 Quiz #1

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Full Name: Solutions Student # :

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Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

[5 marks] (1.) Differentiate $f(x) = \dfrac{1 - xe^x}{x + e^x}$.

$$f'(x) = \frac{(e^x - xe^x)(x+e^x) - (1-xe^x)(1+e^x)}{(x+e^x)^2}$$

$$= \frac{-xe^x - e^{2x} - x^2e^x - xe^{2x} - 1 - e^x + xe^x + xe^{2x}}{(x+e^x)^2}$$

$$= \frac{-e^{2x} - x^2e^x - 1 - e^x}{(x+e^x)^2}$$

$$= \frac{-e^x(x^2+1) - e^{2x} - 1}{(x+e^x)^2}$$

[5 marks] (2.) Show that $x^2 \sin x = \frac{1}{2}$ has a solution on the interval $(\frac{\pi}{6}, \frac{\pi}{2})$.

- use IVT

$f(x) = x^2 \sin x$

$f(x)$ is continuous on $[\frac{\pi}{6}, \frac{\pi}{2}]$

$f(\frac{\pi}{6}) = \frac{\pi^2}{72}$ ≈ 0.1369 $\quad f(\frac{\pi}{2}) = \frac{\pi^2}{4}$ ≈ 2.4649

$\frac{\pi^2}{72} < \frac{1}{2} < \frac{\pi^2}{4} \Rightarrow$ there is some c in $[\frac{\pi}{6}, \frac{\pi}{2}]$ such that $f(c) = 1/2$