Arts & Science 1D06 Quiz #1 25 September 2014

Full Name:________________________________ Student # :______________

TA:________________________________

Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

[10 marks] (1) Consider the function

$$f(x) = e^x \cos(x).$$

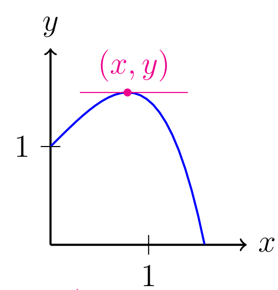
(a) *[5]* Compute the derivative, $f'(x)$.

The function f is a product of two elementary functions: $g(x) = e^x$ and $h(x) = \cos(x)$. We can use the product rule to find the derivative. The product rule says that $(gh)' = g'h + hg'$. Applying this to our function gives:

$$\begin{aligned} f'(x) &= (e^x \cos(x))' \\ &= (e^x)' \cos(x) + e^x(\cos(x))' \\ &= e^x \cos(x) + e^x(-\sin(x)) \end{aligned}$$

$$\boxed{f'(x) = e^x(\cos(x) - \sin(x))}$$

(b) *[5]* Below is the graph of $f(x)$ on the interval $0 \le x \le \frac{\pi}{2}$. What are the (x, y) coordinates of the point at which the slope of the tangent is equal to zero?



The tangent is zero around $(0.8, 1.5)$-ish. We can compute the exact value by setting $f'(x) = 0$. Well, if $e^x(\cos(x) - \sin(x)) = 0$, then either $e^x = 0$ or $\cos(x) - \sin(x) = 0$. The former is impossible, but $\cos(x) - \sin(x) = 0$ is definitely possible!

If $\cos(x) - \sin(x) = 0$ then by moving the $-\sin(x)$ over and we can then divide both sides by $\sin(x)$ we get $\tan(x) = 1$. For what angle is the tangent 1? Well, the tangent of an angle is the slope of the hypotenuse of a right-angle triangle in the unit circle. If the hypotenuse has slope 1, it must be at an angle of $\frac{\pi}{4}$ (draw the picture and check!). Another way to think about this is that if the base and height (i.e. cosine and sine) of a right angle triangle are equal, then the triangle is isosceles and the inside angle must be $\frac{\pi}{4}$. So, $x = \frac{\pi}{4}$ which means that $y = f\left(\frac{\pi}{4}\right) = e^{\frac{\pi}{4}} \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} e^{\frac{\pi}{4}}$.

$$\boxed{\text{The point of zero sloped tangent is therefore } \left(\frac{\pi}{4}, \frac{1}{\sqrt{2}} e^{\frac{\pi}{4}}\right)}$$