Arts & Science 1D06 Quiz #11 Solutions

18 May 2016

Full Name:____ Student # :________________

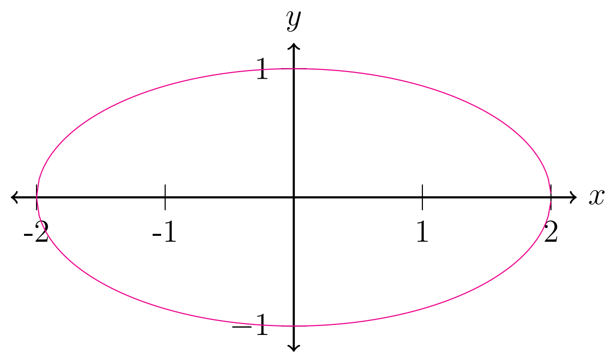
TA:____

Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

[10 marks] (1) This question will deal with the parametric curve defined by

$$x(t) = 2\cos(t), \quad y(t) = \sin(t) \quad , 0 \le t \le 2\pi.$$

(a) *[4]* Draw a rough sketch of the curve.



(b) *[2]* Use the parametric equations to compute $\frac{dy}{dx}$.

$$\frac{dy}{dt} = \cos(t) \text{ and } \frac{dx}{dt} = -2\sin(t), \text{ so } \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = -\frac{\cos(t)}{2sin(t)}.$$

(c) *[3]* Eliminate the parameter t to get an explicit expression for y in terms of x, and use this to compute $\frac{dy}{dx}$ explicitly.

If we write $\frac{x}{2} = \cos(t)$ and $y = \sin(t)$, then $1 = \cos^2(t) + \sin^2(t) = \left(\frac{x}{2}\right)^2 + y^2$. Solving for y and taking the derivative, we get

$$y = \sqrt{1 - \frac{x^2}{4}} \implies \frac{dy}{dx} = -\frac{2x}{4}\frac{1}{2\sqrt{1-\frac{x^2}{4}}} = -\frac{x}{4y}.$$

(d) *[1]* Verify that your answers for (b) and (c) are the same.

Well, if we sub $x = 2\cos(t)$ and $y = \sin(t)$ into the answer from part (c), we get

$$\frac{dy}{dx} = -\frac{2\cos(t)}{4\sin(t)} = -\frac{\cos(t)}{2\sin(t)},$$

which is precisely the answer for (b). Sweet!