Arts & Science 1D06 Quiz #8

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Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

[4 marks] (1) Use the comparison test to decide if the series $\sum_{n=1}^{\infty} \frac{1}{n^n}$ converges.

If we notice that $n^n \geq n^2$ for every natural number n, then we get that $\frac{1}{n^n} \leq \frac{1}{n^2}$. Now since $\sum \frac{1}{n^2}$ is a convergent p-series ($p = 2$), we can conclude by the comparison test that $\sum \frac{1}{n^n}$ is also convergent.

[3 marks] (2) Given that $\frac{1}{n} \leq a_n$ for every n, what can we say about the convergence of $\sum_{n=1}^{\infty} a_n$?

By the comparison test, since $\sum \frac{1}{n}$ diverges, $\sum a_n$ must diverge as well.

[3 marks] (3) Given that $0 \leq b_n \leq \frac{1}{n}$ for every n, what can we say about the convergence of $\sum_{n=1}^{\infty} b_n$?

Given the information we have about b_n, we can't conclude anything about its convergence, since we only know that it is a series of positive terms, bounded above by the harmonic series, which diverges.

$\sum b_n$ could be divergent (e.g. if $b_n = \frac{1}{n}$ for each n) or it could be convergent (e.g. if $b_n = \frac{1}{2^n}$ for each n).