ArtSci 1D06 Calculus
Full year 2015–2016
Instructor: D. Haskell

Winter Midterm – PRACTICE
Thursday 11 February 2016 18:45–20:15

Instructions There are six questions on seven pages. Answer all the questions in the space provided. If you need more paper, ask the invigilator.

NAME:

ID NUMBER:

TUTORIAL DAY AND TIME

Problem	Points
1 [10]	
2 [6]	
3 [6]	
4 [6]	
5 [6]	
6 [6]	
Total [40]	

1) [10 points]

a) State precisely what it means to say that the sequence $\{a_n\}$ diverges.

b) State precisely what it means to say that the sequence $\{a_n\}$ is decreasing.

c) State precisely what it means to say that the series $\sum_{n=0}^{\infty} a_n$ diverges.

d) State precisely what is meant by the interval of convergence of the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$.

e) State precisely what it means to say that the series $\sum_{n=0}^{\infty} a_n$ converges absolutely.

2) [6 points]

a) State the comparison test for convergence of the series $\sum_{n=0}^{\infty} a_n$.

b) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$ converges.

c) Use a partial fraction decomposition to find the exact value of the series in b).

3) [6 points]

a) Use the power series $\dfrac{1}{1-x} = \displaystyle\sum_{n=0}^{\infty} x^n$, for $|x| < 1$ to find a power series representation for the function $f(x) = \dfrac{1}{1+x^2}$, and hence a power series representation for $g(x) = \arctan(x)$.

b) What is the interval of convergence of the series for $g(x)$?

c) Deduce the exact value of the alternating series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$.

4) [6 points] Express the repeated decimal number

$$1.616161\ldots$$

as a fraction by summing an appropriate geometric series.

5) [6 points]

a) Write the formula for the Taylor series around a for a function $f(x)$.

b) Use your answer to a) to find the Taylor series for the function $f(x) = (1 - 3x)^{1/2}$ around 0. (Do not just quote a known Taylor series.)

6) [6 points]

a) State the divergence test.

b) Let $\{a_n\}$ be a decreasing sequence such that $\lim_{n\to\infty} a_n = \frac{1}{2}$. Write $s_m = \sum_{n=1}^{m} a_n$. Find a lower bound for s_m (this will depend on m). Deduce that $\sum_{n=1}^{\infty} a_n$ diverges (thus verifying the divergence test for this example).