ArtSci 1D06 Calculus
Full year 2017–2018
Fall Midterm 1 — Practice version

Instructions There are 5 questions on 4 pages. Answer all the questions in the space provided. You have 50 minutes. If you need more paper, ask the invigilator.

NAME: Solutions

ID NUMBER:

TUTORIAL DAY AND TIME

Problem	Points
1 [6]	
2 [6]	
3 [6]	
4 [6]	
5 [6]	
Total [30]	

1) [6 points] Find the derivatives of the following functions. Do not simplify your answers.

a) $f(x) = \ln(e^{2x} + x)$.

$$f'(x) = \frac{1}{e^{2x} + x} \cdot (2e^{2x} + 1)$$

b) $f(x) = \dfrac{\cos(e^x)}{x + \sin(3x)}$.

$$f'(x) = \frac{(x + \sin(3x))(-\sin(e^x) \cdot e^x) - \cos(e^x) \cdot (1 + 3\cos(3x))}{(x + \sin(3x))^2}$$

c) $f(x) = \arcsin(x^3)$.

$$f'(x) = \frac{1}{\sqrt{1 - (x^3)^2}} \cdot 3x^2$$

2) [6 points] Given $h(x) = f(\sin(x))$, $f(0) = 2$ and $f'(0) = 5$, find $h(\pi)$ and $h'(\pi)$.

$$h(\pi) = f(\sin(\pi)) = f(0) = 2$$

$$h'(x) = f'(\sin(x)) \cdot \cos(x)$$

$$h'(\pi) = f'(\sin(\pi)) \cdot \cos(\pi)$$

$$= f'(0) \cdot (-1)$$

$$= -5.$$

3) [6 points]

a) State the Intermediate Value Theorem.

Suppose f is continuous on $[a,b]$. Let L be any number between $f(a)$ and $f(b)$. There is $c \in [a,b]$ with $f(c) = L$.

b) Show that there exists a number x which is one greater than its cube.

Show there is x with $x = x^3 + 1$. Consider $f(x) = x - x^3 - 1$.

$f(0) = -1$, $f(1) = 1 - 1 - 1 = -1$, $f(-2) = -2 - (-8) - 1 = 5$.

f is continuous everywhere. $f(-2) > 0 > f(0)$. So by the IVT there is $x \in [-2, 0]$ with $f(x) = 0$.

4) [6 points]

a) State the limit definition of the derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

b) Given the function $f(x) = \dfrac{1}{x-3}$, find its derivative f from the definition.

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h-3} - \frac{1}{x\cancel{+h}-3}}{h} = \lim_{h \to 0} \frac{\frac{x \cancel{+h} - 3 - (x+h-3)}{(x \cancel{+h} - 3)(x+h-3)}}{h}$$

$$= \lim_{h \to 0} \frac{1}{h} \cdot \frac{-h}{(x+h-3)(x-3)} = \lim_{h \to 0} \frac{-1}{(x+h-3)(x-3)}$$

$$= \frac{-1}{(x-3)^2}.$$

5) [6 points]

a) Find the linear approximation $L(x)$ to the function $f(x) = x^3$ at $x = 2$.

$f'(x) = 3x^2, \quad f'(2) = 12, \quad f(2) = 8$

$L(x) = f(2) + f'(2)(x-2)$

$= 8 + 12(x-2).$

b) Use the linear approximation to estimate $(2.001)^3$.

$L(2.001) = 8 + 12(0.001)$

$= 8.012$

c) With reference to the graph of $y = x^3$, decide if your estimate is an overapproximation or an underapproximation.

At $x=2$, the tangent line lies under the curve, as can be seen in the graph. So $L(2.001)$ is an underestimate to $f(2.001)$.