MATH 1AA3, Winter term 2006

Practice Problems for Final Exam

Problem 1. Compute the following integrals. If the integrals are divergent, explain why they are divergent.

a) $\int \frac{x+1}{x^2+1} dx$

b) $\int \frac{2x+1}{x^2+2x+2} dx$

c) $\int \frac{x}{x^2+4x+3} dx$

d) $\int \frac{x}{x^2-2x+1} dx$

e) $\int \frac{x-3}{x^2+4x+4} dx$

f) $\int \cos^4(x) dx$

g) $\int \frac{\sqrt{x^2-1}}{x^3} dx$

h) $\int \sqrt{4-x^2} dx$

i) $\int \sqrt{1-4x^2} dx$

j) $\int e^{2x} \cos(x) dx$

k) $\int x^2 \ln(x) dx$

l) $\int \cos(\sqrt{x}) dx$

m) $\int \arctan(\frac{1}{x}) dx$

n) $\int_1^\infty \frac{1}{x^2} dx$

Problem 2. (i) Find the power series centered at 0 representing the following functions.

(ii) Determine the radius of convergence for each power series.

a) $\int e^{x^7} dx$

b) $\int \sin(x^5) dx$

c) $\frac{x^2}{4-x} dx$

d) $\frac{x}{1+x^2} dx$

e) $\frac{1+x}{1-x}$

f) $\int \frac{x^2}{1-x^2} dx$

Problem 3. Find the Maclaurin series for $\arctan(x)$.

Problem 4. Find all the second order partial derivatives of the following functions:

a) $f(x, y) = \frac{xy}{x-y}$

b) $f(x, y) = e^{x \sin(y)}$

Problem 5. Let $f(x, y)$ be defined as $\frac{x^2y^2}{x^4+y^4}$ if $(x, y) \neq (0, 0)$, while $f(0, 0) = 0$.

(i) Find $f_x(x, y)$ and $f_y(x, y)$ for $(x, y) \neq (0, 0)$.

(ii) Find $f_x(0, 0)$ and $f_y(0, 0)$.

(iii) Is f differentiable at $(1, 1)$? Explain.

(iv) Is f differentiable at $(0, 0)$? Explain.

Problem 6. Find the equation of the tangent plane at $(1, 1, 2)$ on the graph of $f(x, y) = x + e^{x-y}$. Find the linear approximation to $f(1.1, 1.2)$.

Problem 7. Consider the triangle with vertices $(-1, 0)$, $(1, 0)$ and $(1, 1)$. Write parametric equations (there will be three sets in all) which describe the motion of a point that moves along the perimeter of this triangle clockwise beginning at $(-1, 0)$ at time $t = 0$ and ending at $(-1, 0)$ at time $t = 3$.

Problem 8. (i) Find the Taylor series centered at 0 for $\ln(1 + x)$.

(ii) For what value of n is the remainder term less than 10^{-5} when the degree n Taylor polynomial for $\ln(1 + x)$ is used to estimate $\ln(1.1)$?

Problem 9. Suppose the function $y = f(x)$ verifies the differential equation $\frac{dy}{dx} = x^2 + y^3$. Use Euler's method with step size 0.25 to estimate $f(1)$ given that $f(0) = -1$.

Problem 10. (i) Sketch the polar curve $r = 1 + \sin(\theta)$.

(ii) Find the area enclosed by the given polar curve in the upper right quadrant.

Problem 11. Find the absolute maximum and minimum of the function $f(x, y) = x^2 + y^2 + x$ on the unit disk:

$$D = \{(x, y) : x^2 + y^2 \leq 1\}.$$

Problem 12. Solve the following differential equations:

(a) $\frac{dy}{dx} = xy + x + y + 1$

(b) $\frac{dy}{dx} = e^{x+y}$

Problem 13. Find the length of the polar curve $r = \sin(\theta) + \cos(\theta)$.