

Student Name and Number:

Math 1C03: Introduction to Mathematical Reasoning

Instructor: Deirdre Haskell

McMaster University Final Exam, 17 April 2007

THIS EXAMINATION PAPER CONTAINS TWO PAGES AND FIVE QUESTIONS, NUMBERED PART I, 1–3, PART II, 1–2. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE EXAMINATION IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF THE INVIGILATOR.

Duration of exam: two hours.

Any calculator is allowed.

Answer all the questions below. Write your solutions in the answer book provided, numbered clearly.

Part I

1) [5 points] Give precise definitions of the following mathematical terms:

- (a) the function $f : A \rightarrow B$ is *injective*
- (b) the relation R on the set A is *transitive*
- (c) b is the *least upper bound* of the set A
- (d) the set A is *countable*

2) [5 points] Define the relation R on the real numbers by $xRy \iff x - y$ is rational.

- (a) Prove that R is an equivalence relation.
- (b) Describe $[0]$, the equivalence class of 0.
- (c) Are there more than 2 equivalence classes? Justify your answer.

3) [5] Let f be a function on the real line whose domain includes the set A .

We say that f is *continuous on A* if for every a in A and for every $\varepsilon > 0$ there is a $\delta > 0$ such that, for all $x \in A$ if the distance between x and a is less than δ then the distance between $f(x)$ and $f(a)$ is less than ε .

We say that f is *uniformly continuous on A* if for every $\varepsilon > 0$ there is a $\delta > 0$ such that for every x and y in A if the distance between x and y is less than δ then the distance between $f(x)$ and $f(y)$ is less than ε .

- (a) State carefully both definitions in symbolic form.
- (b) Explain in your own words the difference between the two properties.
- (b) The function $f(x) = 1/x$ is continuous, but not uniformly continuous on the interval $(0, \infty)$. With $\varepsilon = 1$ illustrate through examples how the definition of uniform continuity fails to be satisfied.

... continued

Student Name and Number:

Part II

1) [20 points] Write a short description (1–2 pages) of one of the Millennium Problems. Your account of it should include the name and an approximate statement of the problem, some background explaining the terminology used in the statement, and some context on why the problem is important.

2) [10 points] Write a short description (1/2–1 page) of one of the student presentations. Your account should focus on the mathematics that was presented, not on the structure of the talk. Indicate clearly which presentation you are describing. You may not choose your own presentation, nor a presentation on the same book that you read.

THE END