Section 3.1 Question 15

Consider the following theorem.

> **Theorem**. Suppose x is a real number and $x \neq 4$. If $\frac{2x-5}{x-4} = 3$ then $x = 7$.

(a) What's wrong with the following proof of the theorem?

Proof. Suppose $x = 7$. Then $\frac{2x-5}{x-4} = \frac{2(7)-5}{7-4} = \frac{9}{3} = 3$. Therefore if $\frac{2x-5}{x-4} = 3$ then $x = 7$.

Answer: What this proves is that if $x = 7$ then $\frac{2x-5}{x-4} = 3$, the converse of the stated theorem. Since $A \rightarrow B$ and $B \rightarrow A$ are different statements, this proof tells us nothing about the truth or falsehood of the theorem in question.

(b) Give a correct proof of the theorem.

Answer: Suppose $x \neq 4$ and $\frac{2x-5}{x-4} = 3$. Since $x - 4 \neq 0$ we can multiply both sides of the equation by it and get:
$2x - 5 = 3x - 12$. That is,
$0 = x - 7$ so
$7 = x$.
Therefore, if $x \neq 4$ and $\frac{2x-5}{x-4} = 3$, then $x = 7$.

Section 3.1 Question 16

Consider the following incorrect theorem.

> **Incorrect Theorem**. Suppose that x and y are real numbers and $x \neq 3$. If $x^2y = 9y$ then $y = 0$.

(a) What's wrong with the following proof of the theorem?

Proof. Suppose that $x^2y = 9y$. Then $(x^2 - 9)y = 0$. <u>Since $x \neq 3$, $x^2 \neq 9$</u> so $x^2 - 9 \neq 0$. Therefore we can divide both sides of the equation $(x^2 - 9)y = 0$ by $x^2 - 9$, which leads to the conclusion that $y = 0$. Thus, if $x^2y = 9y$ then $y = 0$.

Answer: The underlined portion of the incorrect proof is a false implication. $x^2 \neq 9$ tells us that $x \neq 3$, but the implication is not true in reverse.

(b) Show that the theorem is incorrect by finding a counterexample.

Answer: When $x = -3$ and $y \neq 0$, we get $x^2y = 9y$.