Section 3.2 Question 7

Suppose that $x + y = 2y - x$ and x and y are no both zero. Prove that $y \neq 0$.

Answer: Suppose for a contradiction that $y = 0$. Since x and y cannot both be zero (given), we know that $x \neq 0$. However, $x + y = 2y - x$ tells us (through rearranging) that $2x = y$ and since $y = 0$ we conclude that $x = 0$ as well, contradicting what we got above (that $x \neq 0$), therefore our initial assumption that $y = 0$ must have been false.

Section 3.2 Question 12

Consider the following incorrect theorem:

> **Incorrect Theorem.** Suppose that $A \subseteq C$, $B \subseteq C$, and $x \in A$. Then $X \in B$.

(a) What's wrong with the following proof of the theorem?

Proof. Suppose that $x \notin B$. Since $x \in A$ and $A \subseteq C$, $x \in C$. Since $x \notin B$ and $B \subseteq C$, $x \notin C$. But now we have proven both $x \in C$ and $x \notin C$, so we have reached a contradiction. Therefore $x \in B$.

Answer: The underlined sentence in the proof is incorrect, the rest is correct. The underlined sentence claims that $x \notin B$ and $B \subseteq C$ allows us to conclude that $x \notin C$, but this is not the case.

(b) Show that the theorem is incorrect by finding a counterexample.

Answer: There are many appropriate counterexamples.

A very simple one is: $A = \{1\}$, $B = \{2\}$, $C = \{1, 2\}$ and $x = 1$.

A more complicated one: $A = \mathbb{Z}$ (the integers), $B = \mathbb{N}$ (the natural numbers), $C = \mathbb{R}$ (the real numbers) and $x = -1$.

Any situation where $A \not\subseteq B$ (A is not a subset of B) will work.