Section 5.1 Question 11

Suppose $f : A \to B$ and S is a relation on B. Refine a relation R on A as follows:
$R = \{(x, y) \in A \times A | (f(x), f(y)) \in S\}$.

(a) Prove that if S is reflexive, then so is R.

Answer: Assume that S is reflexive. Given any $x \in A$ we need to prove that $(x, x) \in R$.

Proof: Since S is reflexive, we know that $(f(x), f(x)) \in S$ and so, by the definition of R, $(x, x) \in R$.

(b) Prove that if S is symmetric, then so is R.

Answer: Assume that S is symmetric. Given any $(x, y) \in R$ we need to prove that $(y, x) \in R$.

Proof: Since $(x, y) \in R$, we know that $(f(x), f(y)) \in S$ by the definition of R. Since S is symmetric this tells us that $(f(y), f(x)) \in S$ as well and so the definition of R then gives us that $(y, x) \in R$.

(c) Prove that if S is transitive, then so is R.

Answer: Assume that S is transitive. Given any $(x, y) \in R$ and $(y, z) \in R$ we need to prove that $(x, z) \in R$.

Proof: Since $(x, y) \in R$ and $(y, z) \in R$ we know that $(f(x), f(y)) \in S$ and $(f(y), f(z)) \in S$. Transitivity of S then tells us that $(f(x), f(z)) \in S$ and so $(x, z) \in R$ by definition.