McMaster University Math 1XX3 Winter 2013
Final Exam — PRACTICE version

Duration: 3 hours

Instructor: Dr. D. Haskell

Name: ____

Student ID Number: ______________

This test paper is printed on both sides of the page. There are 12 question on 12 pages. You are responsible for ensuring that your copy of this test is complete. Bring any discrepancies to the attention of the invigilator. More paper for rough work is available from the invigilator.

Instructions

(1) Only the standard McMaster calculator is allowed.
(2) All answers must be written in the space following the question. If you write an answer on scratch paper, indicate **clearly** where to find your answer.

This PRACTICE version of the midterm is intended to give you an idea of the format, approximate length and approximate difficulty of the actual midterm. There is no guarantee as to the actual length and difficulty of the actual exam. In particular, the actual midterm will NOT be “just the same with the numbers changed”.

1) [10 points]

a) State the Integral Test for convergence of the series $\sum_{n=0}^{\infty} a_n$.

b) State the Fundamental Theorem of Calculus Part I.

c) Find $\int_1^2 x^2 \, dx$.

d) Find cartesian coordinates for the point $(2, \pi/3)$ in polar coordinates.

e) Sketch the slope field for the differential equation $\frac{dy}{dx} = y$.

2) [10 points]

a) Sketch the graph of $x^2 + 4y^2 = 1$.

b) Give an example of a sequence which is bounded but not monotonic.

c) Is the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ convergent or divergent. Justify your answer briefly.

d) Suppose $x(t)$, $y(t)$ are two interacting populations described by the differential equations

$$\frac{dx}{dt} = 0.01x - 0.5xy \qquad \frac{dy}{dt} = -0.01y + 0.002xy\ .$$

Which variable represents the predator population, and which is the prey? Explain briefly.

e) Find $\sum_{n=1}^{100} 2$.

3) [8 points] Consider the curve given in parametric form by the equations $x = \theta - \sin(\theta)$, $y = 1 - \cos(\theta)$.

a) Complete the table of values.

θ	0	$\pi/2$	π	$3\pi/2$	2π	$5\pi/2$	3π	$7\pi/2$	4π
x									
y									

b) Find the points in the interval $0 \leq \theta \leq 4\pi$ at which the curve has a horizontal tangent line.

c) Find the points in the interval $0 \leq \theta \leq 4\pi$ at which the curve has a vertical tangent line.

d) Sketch the curve.

4) [8 points]

a) Find the Taylor series around 0 for the function $f(x) = \cos(x)$.

b) Determine the number of terms of the Taylor series that are needed in order to estimate $\cos(0.1)$ with an error of less than 10^{-4}.

c) Use your Taylor series from part a) to find $\cos(0.1)$ with an error of less than 10^{-4}.

5) [8 points] Find the indefinite integral $\int \frac{1}{(x-3)(x-2)}\,dx$.

6) [8 points] Consider the power series $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{3^n(n+1)}$.

a) Find the radius of convergence.

b) Find the interval of convergence.

7) [8 points] Test the series $\sum_{n=2}^{\infty} \frac{8^n}{2+9^n}$ for convergence. State any test you use, and be careful to check its hypotheses.

9) [8 points] Solve the differential equation $x\ln(x)\dfrac{dy}{dx} + y = xe^x$.

10) [8 points] Find the improper integral $\int_1^\infty (1 - \frac{1}{x}) e^{\ln(x) - x}\ dx$.

11) [8 points] Find the volume of the solid formed by rotating the graph of $y = \sin(x)$ from $x = 0$ to $x = \pi$ around the line $y = 1$.

12) [12 points]

a) What can you say about solutions to the differential equation $\frac{dy}{dx} = x^2 + y^2$ just by looking at the differential equation?

b) Let C_1 be the curve defined by the equation $r = 3$, C_2 be the curve defined by the equation $x^2 + y^2 = 9$ and C_3 be the curve defined by the pair of equations $x = 3\sin(2t), y = 3\cos(2t)$. Compare C_1, C_2, C_3.

c) Suppose $\sum a_n$ is a convergent sum of positive terms. Does it follow that $\sum(-1)^n a_n$ converges?