SHORT MAPLE MANUAL

Algebraic Operations, Functions and Constants

Elementary algebraic operations are denoted by `+`,`-`,`*` and `/`. The sign for exponentiation is `^`. Only one type of parentheses is allowed, namely the round parentheses `( )`. If a parenthesis does not have its mathcing pair Maple will generate an error message.

The symbol for the absolute value is `abs()`. For example, the expressions $|x-6|$ and $||f(x)|-|x||$ have to be typed as `> abs(x-6);` and `> abs(abs(f(x))-abs(x));`

The symbol `!` is used for factorials. To find the numeric value of 1000!, type `> 100!;`

Constants: π is denoted by `Pi` (uppercase `P`) and e, the basis of natural logarithms, is obtained by executing the command `exp(1)`. To type ∞ use `infinity`.

The symbol for powers is `^`. To type x^3, $t^{6/7}$ and $z^{-2.11}$ use `x^3`, `t^(6/7)` and `z^(-2.11)`. The square root function $\sqrt{x}$ can be input as `sqrt(x)` or as `x^(1/2)`. The symbol for $\sqrt[n]{x}$ is `surd(x,n)`. For example, $\sqrt[3]{2x}+4\sqrt[5]{x^2}$ is entered as

```
> surd(2x,3)+4*surd(x^2,5);
```

or, of course, as

```
> (2x)^(1/3)+4*(x^2)^(1/5);
```

The symbols for trigonometric functions are `sin(x)`, `cos(x)`, `tan(x)`, `cot(x)`, `sec(x)` and `csc(x)`. The variable `x` is in radians. Inverse trigonometric functions are denoted by `arcsin(x)`, `arctan(x)`, etc. The exponential function e^x is denoted by `exp(x)` and its inverse, the natural logarithm function $\ln x$ by `ln(x)`. The symbols for hyperbolic functions are `sinh(x)`, `cosh(x)`, `tanh(x)`, etc.

The symbol % refers to the result of the previous computation. For example,

```
> 4*5;
```

will return 20; the command

```
> %/10;
```

will return 2 (since % is equal to 20). But now

```
> %/10;
```

will return 1/5, since % is equal to 2.

Evaluating Numeric Expressions

Maple always tries to return the answer in the most accurate form. That is the reason why, for example, we see the response `arctan(2)` to the command

```
> solve(tan(x)=2,x);
```

that asks for the solution of the equation $\tan x = 2$. To obtain a decimal (numeric) approximation of $\arctan 2$, we use either `> evalf(%);` immediately after the `solve` command or combine the two commands into one: `> evalf(solve(tan(x)=2,x));` The last command can be abbreviated as `> fsolve(tan(x)=2,x);`

If more digits are needed, we use

`evalf(EXPRESSION,NUMBER OF DIGITS)`

The default value is 10. To display the first 45 digits of the number $2^{23}/11^5$, we use

```
> 2^23/11^5;evalf(%,45);
```

or, joined together in a single command

```
> evalf(2^23/11^5,45);
```

The command `> evalf(exp(1),1000);` will return 1000 digits (hence 999 decimals) of the basis $e = 2.71828...$ of natural logarithms.

Maple will sometimes respond with a symbolic expression, although we have assigned numeric values to all symbols involved. If this is the case, execute the command `evalf(%)` immediately after the response that should have been evaluated. For example, after defining the function

```
> f := x -> exp(x)+sin(x)+x^2+1;
```

we ask for its value at $x = 2$:

```
> subs(x=2,f(x));
```

but the response $e^2 + \sin 2 + 5$ is probably not what we were looking for. To obtain a numeric value, we execute `> evalf(%);` immediately after `subs` command. Alternatively, use `> evalf(subs(x=2,f(x)));`

To simplify the symbolic expression we have obtained, we use the command

`simplify(EXPRESSION)`

where EXPRESSION can be replaced by % if we want to tell Maple to simplify the result we have just obtained.

Assigning Values

The command

`SYMBOL := VALUE`

assigns the VALUE to the SYMBOL. For example, `> a := 23;` assigns the numeric value 23 to the constant a. The assignment

```
> m := fsolve(4*m^3-11=0,m);
```

assigns the decimal approximation 1.401019665 of the solution of the equation $4m^3 - 11 = 0$ to the symbol m.

Functions

The command

`FUNCTION := VARIABLE -> RULE`

defines a function: FUNCTION is the name of the function and VARIABLE defines its variable.

For example, to define a function $f(x) = 3x - \tan x$, we use

```
> f := x -> 3*x-tan(x);
```

The command `> h := u -> (u^2-u)/(u+u^3-1);` defines the function $h(u) = \dfrac{u^2 - u}{u + u^3 - 1}$
To find the value of $f(x)$ at $x = 2$, we can use either

```
> evalf(f(2));
```

or the `subs` command: `> subs(x=2,f(x));`
Likewise, to compute the value of the previously defined function $h(u)$ at $u = -0.2$, we can use either `> evalf(h(-0.2));` or `> evalf(subs(u=-0.2,h(u)));` (remember that we have to use `evalf` command to ask for a decimal number as a result; without it, Maple might return some algebraic expression or will just retype our command).

Every time we refer to a function, we have to keep the independent variable, i.e., we need `f(x)` and not just `f`. Functions can be used to define other functions. We have already defined $f(x)$; now, to define the function $g(x) = 2f(x)^2 - \cos(f(x))$, we execute the command

```
> g := x -> 2*f(x)^2-cos(f(x));
```

Notice that we always use `f(x)` (and not just `f`) when refering to a function.

`subs` command becomes useful when one has to compute the value of a derivative of a function. For example, to compute $f'(3.1)$ we use

```
> evalf(subs(x=3.1,diff(f(x),x)));
```

and to compute $f''(3.1)$ we use

```
> evalf(subs(x=3.1,diff(f(x),x$2)));
```

Plotting Functions

To plot the graph of a function, use

plot(FUNCTION,VARIABLE=INTERVAL)

where INTERVAL defines the interval where the plot will be constructed. For example, to obtain the plot of the function $f(x) = x^2 \sin x - \cos(3x)$ on the interval $[0, \pi]$ execute

```
> plot(x^2*sin(x)-cos(3*x),x=0..Pi);
```

To see the plot of $h(s) = \arcsin(s^2) + s$ on $[0, 1]$, you can either execute

```
> plot(arcsin(s^2)+s,s=0..1);
```

or, define the function first

```
> h := s -> arcsin(s^2)+s;
```

and then plot it

```
> plot(h(s),s=0..1);
```

The command

```
> plot(diff(h(s),s),s=0..1);
```

will plot the graph of $h'(s)$ over $[0, 1]$.

The way Maple draws the graph is the following: it plots a lot of points (as we would do: by choosing x and computing the corresponding value $y = f(x)$) and then connects them with line segments. The command

```
> plot(x^2*sin(x)-cos(3*x),x=0..Pi,style=point);
```

will draw the points only, without the line segments connecting them. If you need a graph that is more precise, use `numpoints` increase the number of points which Maple uses to create the plot (the default value could be 25 or 49 or 54). For example, the command

```
> plot(x^2*sin(x)-cos(3*x),x=0..Pi,numpoints=100);
```

will produce the plot of the function over $[0, \pi]$, with the values of the function computed at 100 points. Increasing `numpoints` is one way of obtaining a (more) precise plot of the function. The other way is to "zoom-in" the graph, i.e. to plot it over a smaller interval.

It is possible to plot more functions in the same coordinate system (but it has to be done over the same interval). The command is

```
plot({FUNCTION,...,FUNCTION},VARIABLE=INTERVAL)
```

For example, the command

```
> plot({x^2-x,3*x-2},x=-3..1);
```

will plot both $x^2 - x$ and $3x - 2$ on the interval $[-3, 1]$. Unfortunately, Maple does not label the graphs, and you should be able to determine which is which. The commands

```
> f := x -> sin(x)+2*cos(x);
```

and

```
> plot({f(x),f(x)+3,4*f(x)},x=-Pi..Pi);
```

will plot the functions $f(x) = \sin x + 2\cos x$, $f(x) + 3 = \sin x + 2\cos x + 3$ and $4f(x) = 4\sin x + 8\cos x$ on $[-\pi, \pi]$, in the same coordinate system.

Similarly,

```
> plot({f(x),diff(f(x),x)},x=-Pi..Pi);
```

will give the plot of the function $f(x)$ and its first derivative $f'(x)$.

Once the plot interval is selected, Maple chooses the range on the y-axis so that the graph fits the window. But you can specify the range on the y-axis, if needed. The command is

```
plot(FUNCTION,VARIABLE=INTERVAL,RANGE)
```

For example,

```
> plot(sin(x)+2,x=0..2*Pi,y=2..4);
```

or

```
> plot(sin(x)+2,x=0..2*Pi,y=0..3);
```

or

```
> plot(sin(x)+2,x=0..2*Pi,y=-1..5);
```

will plot the graphs of the same function, which appear different, due to different ranges on the y-axis.

Solving Equations

The command that attempts to solve an equation is

`solve(LEFT SIDE=RIGHT SIDE,UNKNOWN)`

if we ask for the algebraic (symbolic) expression as an answer, or

`fsolve(LEFT SIDE=RIGHT SIDE,UNKNOWN)`

if we need a numeric approximation of the solution(s). For example,

```
> solve(a*x^2+b*x+c=0,x);
```

will return $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and `> solve(x^2=2,x);` will return $\sqrt{2}$ and $-\sqrt{2}$.

On the other hand, `> fsolve(x^2=2,x);` will respond with numeric approximations 1.414213562 and −1.414213562 of $\sqrt{2}$ and $-\sqrt{2}$. Sometimes, `solve` command will not return anything. For example, `> solve(cos(x)=x,x);` will give no meaningful answer, whereas `> fsolve(cos(x)=x,x);` gives the solution 0.7390851332 in the decimal form.

We can also tell Maple where to look for solutions. The command

`fsolve(LEFT SIDE=RIGHT SIDE,UNKNOWN=INTERVAL)`

will look for the solution(s) in the given interval. For example,

```
> fsolve(cos(x)=0.1*x,x=0..1);
```

will give the solution of $\cos x = 0.1x$ that belongs to the interval $[0, 1]$.

There are situations when Maple can not find all solutions to a given equation. If this is the case, the combination of the graph of the function and `fsolve` commands with specified interval (that is read off from the graph) might be very useful.

Differentiation

The command that computes the first derivative of a function is

`diff(FUNCTION,VARIABLE)`

where VARIABLE denotes the variable with respect to which the derivative is computed. To compute the derivative of $\sin(3\cos x) + 3.2\ln(x + 4)$ we use

```
> diff(sin(3*cos(x))+3.2*ln(x+4),x);
```

Remember that you can use `simplify(%)` to try to convert the result to a more useful form. We can define the function first, and then compute its derivative; the commands

```
> F := u -> u^3+2*u^2-exp(u)+5;
```

and

```
> diff(F(u),u);
```

will return the derivative of $F(u) = u^3 + 2u^2 - e^u + 5$. It does not suffice to type `F` for the function: Maple needs `F(u)`.

The command for the nth derivative of a function is

```
diff(FUNCTION,VARIABLE$N)
```

For example, the commands

```
> f := x -> arcsin(x^2+x)+sin(cos(x));
```

and

```
> diff(f(x),x$3);
```

will define the function $f(x) = \arcsin(x^2+x)+\sin(\cos x)$ and compute its third derivative.

Derivative of a function is again a function, so all relevant commands apply. For example, the command

```
> plot(diff(f(x),x),x=0..4);
```

will plot the derivative of the (previously defined) function $f(x)$ over the interval $[0, 4]$. The result of the execution of

```
> plot({f(x),diff(f(x),x),diff(f(x),x$2)},x=-2..-1);
```

is the plot of the function $f(x)$ and its first and second derivatives (on the same graph) over the interval $[-2, -1]$.

To evaluate the value of a derivative of a function, we use

```
subs(VARIABLE=VALUE,FUNCTION)
```

The command

```
> evalf(subs(x=1.5,diff(f(x),x$2)));
```

will return the numeric value (that is the reason why we added `evalf`) of the second derivative of $f(x)$ at $x = 1.5$. Similarly,

```
> subs(x=a, diff(f(x),x$3));
```

will give the symbolic expression that is the value of the third derivative of $f(x)$ at $x = a$. To compute the slope m of the tangent line to the graph of $h(x) = \arctan x$ at $x = \pi$, we execute the command

```
> h := x -> arctan(x);
```

to define the function $h(x)$, and then

```
> m := subs(x=Pi,diff(h(x),x));
```

Integration

To find an indefinite integral of a function, we execute the command

```
int(FUNCTION,VARIABLE)
```

where FUNCTION is the integrand and VARIABLE the variable of integration. For example, to find

$$\int (\arctan x + e^x + 7x - 2)\ dx,$$

we use

```
> int(arctan(x)+exp(x)+7*x-2,x);
```

The definite integral is computed similarly:

int(FUNCTION,VARIABLE=INTERVAL)

where INTERVAL represents the interval of integration, in the form `a..b` (`a` is a real number or $-\infty$ (Maple's symbol for ∞ is `infinity`), and `b` is a real number or ∞). For example, to find

$$\int_2^{3.4} (t + 1.33 \ln t)\, dt,$$

we use

```
> int(t+1.33*ln(t),t=2..3.4);
```

The command

```
> int(exp(-x^2),x=-infinity..infinity);
```

returns the value of the definite integral

$$\int_{-\infty}^{\infty} e^{-x^2}\, dx.$$

We can define the function first, and then integrate it. For example, to compute

$$\int_1^5 \frac{x^2+1-x}{3x^4-x^2}\, dx,$$

we execute

```
> f := x -> (x^2+1-x)/(3*x^4-x^2);
```

and then

```
> evalf(int(f(x),x=1..5));
```

To define the function

$$g(x) = \int_0^{\sin x} (t^2+1)\, dt$$

we execute

```
> g := x -> int(t^2+1,t=0..sin(x));
```

Now we can do anything we do with any other function: for example, to evaluate its value at $x = \pi/2$ we use

```
> evalf(g(Pi/2));
```

or

```
> evalf(subs(x=Pi/2,g(x)));
```

We can plot $g(x)$

```
> plot(g(x),x=0..2*Pi);
```

(we chose the plot interval to be $[0, 2\pi]$), compute its derivative

```
> diff(g(x),x);
```

or integrate it (say, over the interval $[0, 1]$)

```
> evalf(int(g(x),x=0..1));
```