Math 3TP3 Truth and Provability Term 1 Autumn 2014–2015
Assignment 5 due November 14, 2014

1) Find the gödel numbers of the six non-logical axioms of PA (the list of axioms is on p. 92 of the text, and the list of basic gödel numbers is on p. 137). Here is the first one, to get you started.

$$\begin{aligned}\ulcorner \mathrm{Ax1} \urcorner &= \ulcorner \forall x(0 \neq Sx) \urcorner \\ &= \ulcorner \forall x \neg (0 = Sx) \urcorner \\ &= 2^{11} \cdot 3^2 \cdot 5^1 \cdot 7^{17} \cdot 11^{21} \cdot 13^{15} \cdot 17^{23} \cdot 19^2 \cdot 23^{19}\end{aligned}$$

   There is no need to multiply out!

2) Find $\mathrm{Diag}(\varphi)$, where φ is the formula $y + S0 = SS0$. Point out the difficulties with explicitly writing down $\ulcorner \mathrm{Diag}(\varphi) \urcorner$.

3) Determine which of the following statements are true and which are false. Explain your reasoning.
   (a) $\mathrm{Var}(2^3)$
   (b) $\mathrm{Termseq}(2^{2^2} \cdot 3^{2^{23} \cdot 3^2} \cdot 5^{2^4} \cdot 7^{2^{23} \cdot 3^2 \cdot 5^{25} \cdot 7^4})$
   (c) $\mathrm{Term}(2^{23} \cdot 3^2 \cdot 5^{25} \cdot 7^4)$
   (d) $\mathrm{Form}(2^{15} \cdot 3^{23} \cdot 5^2 \cdot 7^{25} \cdot 11^4)$
   (e) $\mathrm{Prf}(2^{2^{11} \cdot 3^2 \cdot 5^{17} \cdot 7^2 \cdot 11^{25} \cdot 13^{21} \cdot 17^{15} \cdot 19^2 \cdot 23^{19}} \cdot 3^{2^{23} \cdot 3^{21} \cdot 5^{25} \cdot 7^{21} \cdot 11^{15} \cdot 13^{23} \cdot 17^{21}}, 2^{23} \cdot 3^{21} \cdot 5^{25} \cdot 7^{21} \cdot 11^{15} \cdot 13^{23} \cdot 17^{21})$

4) Now create your own example: find m and n such that $\mathrm{Formseq}(m)$ is true and $\mathrm{Form}(n)$ is false. Justify.