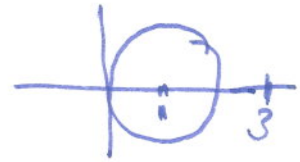
Priestley Chapter 11 Some Solutions

11·1 (i) $\displaystyle\int_{\gamma(1;1)} \frac{1}{z-3}\,dz = 0$ by the Cauchy theorem:

$\dfrac{1}{z-3}$ is holomorphic on



(ii) $\displaystyle\int_{\gamma(i;4)} \frac{1}{(z-3)^3}\,dz = 0$ by the fundamental integral (3 is inside the circle, as $|3-i| = \sqrt{10} < 4$)

(iii) $\displaystyle\int_{\gamma(0;1)} z\,|z|^4\,dz = \int_0^{2\pi} e^{it}\, 1^4\, i e^{it}\,dt = 0$ by ~~the~~ ~~fundamental~~ direct calculation.

(iv) $\displaystyle\int_{\gamma(1;1)} \frac{1}{1+e^z}\,dz = 0$ by Cauchy's theorem — the poles of $\dfrac{1}{1+e^z}$ are at $z = \pi i + 2\pi i k$, which are not inside the given circle.

11·2 (a) $\gamma(0;2)$ $\quad z = 2e^{it} \quad 0 \le t \le 2\pi$.

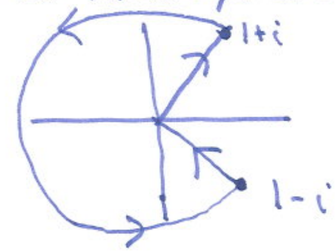
(i) $\displaystyle\int_\gamma \bar{z}\,dz = \int_0^{2\pi} 2e^{-it}\, 2ie^{it}\,dt = 8\pi i$.

(ii) $\displaystyle\int_\gamma \frac{1}{z-1}\,dz = 2\pi i$ by the fundamental integral, as 1 is inside γ.

(iii) $\int_\gamma z^5 \sin^3(z)\,dz = 0$ by Cauchy's theorem, as f is holomorphic on the disc.

(iv) $\int_\gamma \sec^2(z)\,dz = 0$ by the fundamental theorem of calculus
(Note that $\sec^2(z)$ is not holomorphic inside γ)

(b) γ is the contour



(i) $\int_\gamma \bar{z}\,dz = 3\pi + i$ (three contour integrals).

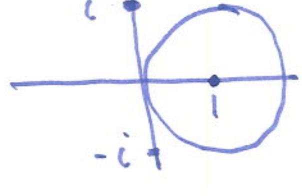
(ii) $\int_\gamma \frac{1}{z-i}\,dz = 0$ as $\frac{1}{z-i}$ is holomorphic inside γ

(iii) $\int_\gamma z^5 \sin^3(z)\,dz = 0$ as f is holomorphic inside and on γ.

(iv) $\int_\gamma \operatorname{cosec}^2(z)\,dz = 0$ ~~as~~ by the fundamental theorem of calculus.

11.3 $f(z) = \frac{1}{1+z^2} = \frac{1}{(z-i)(z+i)}$

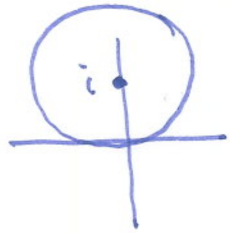
(i) $\gamma(1;1)$



$\pm i$ not inside γ

$\int_\gamma f(z)\,dz = 0$ by Cauchy's theorem

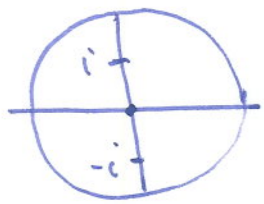
(ii) $\gamma(i;1)$



i inside γ
$-i$ outside γ

$$\int_\gamma f(z)\,dz = \int \frac{1/(z+i)}{z-i}\,dz = \left(\frac{1}{z+i}\right)\Big|_{z=i} \cdot 2\pi i = \pi.$$

(iv) $\gamma(0;2)$



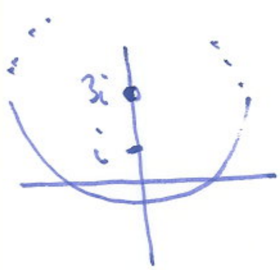
$i, -i$ both inside γ.

$$\int_{\gamma(0;2)} \frac{1}{(z-i)(z+i)}\,dz = \frac{1}{2i}\int_{\gamma(0;2)} \frac{1}{z-i}\,dz - \frac{1}{2i}\int_{\gamma(0;2)} \frac{1}{z+i}\,dz$$

$$= \frac{1}{2i}\,2\pi i - \frac{1}{2i}\,2\pi i$$ by the fundamental integral

$$= 0.$$

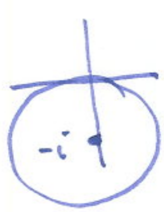
(v) $\gamma(3i;\pi)$



~~γ con~~
i inside γ
$-i$ outside.

As for (ii).

(iii) $\gamma(-i;1)$



$-i$ inside γ
i outside γ

$$\int_\gamma f(z)\,dz = \int \frac{\frac{1}{z-i}}{z+i}\,dz = 2\pi i\left(\frac{1}{z-i}\right)\Big|_{z=-i} = -\pi.$$