Homework 1: Algebra, geometry and topology in the complex plane
due 8:30 17 January 2011

1) Does $z^2 = |z|^2$? If so, prove the equality. If not, decide for what values of z it is true (and prove your answer, of course).

2) Find the subsets of the Riemann sphere that correspond to the real and imaginary axes of the complex plane.

3) Problem 2.14 on p. 28 of Priestley.

4) For any point $z_0 \in \mathbb{C}$, show that $\{z_0\}$ is closed. Hence show that the complement of any finite number of points in $\mathbb{C}$ is an open set.

5) For each of the following subsets of $\mathbb{C}$ decide if it is connected. If not, describe its components.

- $X = \{z \in \mathbb{C} : |z| \leq 1\} \cup \{z \in \mathbb{C} : |z - 2| \leq 1\}$
- $X = \mathbb{C} \setminus (A \cup B)$, where $A = \{z \in \mathbb{C} : \text{re}(z) \in (0, \infty) \text{ and } \text{im}(z) = 0\}$, and $B = \{z = \theta e^{i\theta} : 0 \leq \theta < \infty\}$.