Homework 2: Complex functions: differentiation, power series, examples

due 8:30 1 February 2011

1) Let $f : \mathbb{C} \to \mathbb{C}$ be a function which is holomorphic on a region G. Suppose that the function $\mathrm{re}(f)$ is constant on G. Prove that f is constant on G.

2) Let $f(z) = \dfrac{1}{1-z}$, $g(z) = \dfrac{1}{z(z+2)}$. Write each of f and g as power series, first in powers of $z+1$ and then as powers of $z-i$. In each case, determine the values of z on which the series converge absolutely.

3) Describe the sets on which the following functions are holomorphic and compute their derivatives.

(1) $f(z) = e^{1/z}$

(2) $f(z) = \exp\big(\dfrac{1}{1-az}\big)$, $a \in \mathbb{C}$

(3) $f(z) = \dfrac{\sin(z)}{z}$

4) Priestley 7.10