Homework 3: Integration along paths, Cauchy's theorem
due 8:30 15 February 2011

1) Calculate the following contour integrals.

(i) $\int_\gamma \text{im}(z)\, dz$, where γ is the upper half of the unit circle, traced counterclockwise.

(ii) $\int_\gamma \bar{z}\, dz$, where γ is the unit circle, traced counterclockwise.

(iii) $\int_\gamma (\text{re}(z)^2 - \text{im}(z)^2)\, dz$, where γ is the straight line from 0 to i.

2) Let f be a function which is holomorphic everywhere in a region G except at one point $a \in G$. Suppose there is a number M and an open ball D around a such that for all $z \in D$, $|f(z)| \leq M$. Use the estimation theorem and the deformation theorem to prove that $\int_\gamma f(z)\, dz = 0$ for any simple closed curve γ in G containing the point a.

3) Evaluate the following integrals.

(i) $\int_\gamma (z^3 + 3)\, dz$, where γ is the upper half of the circle radius 2 centered at the origin.

(ii) $\int_\gamma (z^3 + 3)\, dz$, where γ is the circle radius 2 centered at the origin.

(iii) $\int_\gamma e^{1/z}\, dz$, where γ is the circle of radius 2 centered at $3 - 2i$.

4) Find all possible values of $\int_\gamma \frac{dz}{z^2 + z + 1}\, dz$ for γ a simple closed curve.