Homework 6: Cauchy's residue theorem and applications

due 8:30 5 April 2011

1) Find all the singularities of the following functions. Classify them, and compute the residues at those points.

(a) $f(z) = \dfrac{3}{z^2 + 2z + 1}$

(b) $f(z) = \dfrac{1}{z^2 \sinh(z)}$

2) Suppose f_1 and f_2 have residues r_1 and r_2 at a respectively. Show that the residue of $f_1 + f_2$ at a is $r_1 + r_2$.

3) Use the residue theorem to evaluate the following integrals.

(a) $\displaystyle\int_{\gamma(0;8)} \tan(z)\, dz$

(b) $\displaystyle\int_{\gamma(0;5)} \frac{5z - 2}{z(z-1)}\, dz$

4) Use the residue theorem to evaluate the following real integrals.

(a) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1 + x^{2n}}\, dx$

(b) $\displaystyle\int_{1}^{\infty} \frac{1}{x\sqrt{x^2 - 1}}\, dx$