Valued Fields and Elimination of Imaginaries

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1. Introduction
2. Valued Fields
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Definable Sets

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By quantifier elimination, the definable sets are precisely the boolean combinations of algebraic varieties (constructible sets).

RCF real closed fields
By quantifier elimination, the definable sets are precisely the semialgebraic sets.

Philosophical position statement

In order to understand a theory properly, we need to express it in a language in which we have:

Elimination of quantifiers

algebraic, geometric and topological properties of sets are much easier to observe if the sets are defined without quantifiers.

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Model-theoretically, quotients are not so amenable to study.

Definition of imaginaries

Let M be a structure in a language L, and let E be a $\emptyset$-definable equivalence relation on M: $E \subset M^n \times M^n$. Then for any $a \in M^n$,

$$[a]_E = \{y \in M^n : aEy\}$$

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Definition

An equivalence class of a $\emptyset$-definable equivalence relation is an imaginary of the structure.

Definable representatives

If every equivalence class of E had a representative a_E which could be chosen definably, then $\{a_E\}$ would be a definable set in M which corresponds to M^n/E.

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More generally, in order for there to be a definable set in M which corresponds to M^n/E it suffices for there to be a definable function $f : M^n \to M^m$ such that $f(a) = f(b)$ if and only if aEb.

The property of eliminating imaginaries

Definition

Let T be a complete theory in a language L (with at least two constant symbols). T is said to eliminate imaginaries if, for every formula which defines an equivalence relation E there is a $\emptyset$-definable function f_E such that in any model M of T and for every $a, b \in M^n$, $f(a) = f(b)$ if and only if aEb.

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- We will call $c = f_E(a) \in M^m$ a code for $[a]_E$.
- Then c is definable from $[a]_E$, and $[a]_E$ is definable from c.

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1) Elimination of quantifiers

algebraic, geometric and topological properties of sets are much easier to observe if the sets are defined without quantifiers.

2) Elimination of imaginaries

all the above properties of the definable sets carry over to their quotients.

Theories which eliminate imaginaries

Algebraically Closed Fields eliminate imaginaries

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Real Closed Fields eliminate imaginaries

The proof follows from the cell decomposition theorem. An equivalence class in one variable is a finite union of intervals; the endpoints can be used to code the class. In more variables, one uses Skolem functions.

Theories which do not eliminate imaginaries

Theories of valued fields do not eliminate imaginaries, at least not in the usual language.

Definition of a valuation

Definition

A valuation is a map $v : K \to \Gamma \cup \{\infty\}$ from a field K to an ordered group Γ such that for all $a, b \in K$

$$\begin{aligned} v(ab) &= v(a) + v(b) \\ v(a+b) &\geq \min\{v(a), v(b)\} \\ v(0) &= \infty \end{aligned}$$

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valuation ring $\mathcal{O}_K = \{x \in K : v(x) \geq 0\}$
maximal ideal $\mathcal{M} = \{x \in K : v(x) > 0\}$
residue field $\mathcal{O}_K/\mathcal{M} = k$

Example of a valuation

The standard example to keep in mind is a measure of divisibility by a given prime p.

For $x \in \mathbb{Q}$, write $x = p^r(s/t)$. Then $v(x) = r$.

The valuation gives rise to a metric on $\mathbb{Q}$, and the field of p-adic numbers, $\mathbb{Q}_p$, is the completion of $\mathbb{Q}$ with respect to this metric.

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The construction of $\mathbb{Q}_p$ is analogous to that of $\mathbb{R}$, and so one would hope that model-theoretically the theories of the two fields would have similar properties.

Examples of complete theories of valued fields

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pCF p-adically closed fields, for a fixed prime p

The axioms say that K is a valued field and

- $v(p)$ is minimal positive
- the residue field is $\mathbb{F}_p$, the finite field with p elements
- Hensel's lemma is satisfied (an existential closure condition).

The standard model is $\mathbb{Q}_p$.

Examples of complete theories of valued fields

ACVF algebraically closed valued fields

The axioms say that K is a valued field and is algebraically closed. It follows that the residue field k is also algebraically closed, and the value group Γ is divisible. For a complete theory, the field and residue field characteristic also need to be specified. Examples include:

- $\mathbb{Q}_p^{\mathsf{alg}}$ in characteristic $(0, p)$
- $\mathbb{C}[[t]]^{\mathsf{alg}}$ in characteristic $(0, 0)$
- $\mathbb{F}_p[[t]]^{\mathsf{alg}}$ in characteristic (p, p)

Examples of complete theories of valued fields

RCVF real closed valued fields

K is a field with both a valuation and an ordering, and such that every odd degree polynomial has a root. The standard example is $\mathbb{R}(t^{\mathbb{Q}})$.

Elimination of quantifiers

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in the language of fields with a family of predicates P_n which express that an element has an nth root (Macintyre, 1976).

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in the language of fields with a predicate for the valuation ring (Robinson 1956).

RCVF eliminates quantifiers

in the language of ordered fields with a predicate for the valuation ring (Cherlin-Dickmann 1983).

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An infinite definable subset of the field in any of the above theories has interior in the valuation topology.

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Compare with quantifier elimination:

the theory of real closed fields does not eliminate quantifiers in the pure field language; we have to add a predicate for the ordering.

Failure of elimination of imaginaries

Question

What do we need to add to the language in order for theories of valued fields to eliminate imaginaries, without losing quantifier elimination?

Two approaches

Approach 1 Increase the language by adding definable functions to do some of the coding.

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Theorem

Write the theory of p-adically closed valued fields in a language which has definable representatives for the equivalence relations $v(x) = v(y)$ *and* $v(x - y) \geq 0$*. Then the statement that the theory eliminates imaginaries is independent of ZFC (Macintyre and Scowcroft, 1993).*

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Approach 2 Add sorts to the language; that is; work in a fragment of M^{eq}.

M^{eq}

Definition

Let M be an L structure. For each $\emptyset$-definable equivalence relation E on M^n for some n, add a sort S_E for M^n/E. Also add a function $f_E : M^n \to S_E$ given by $f(x) = [x]_E$. Then M^{eq} is the resulting many-sorted structure and $T^{\mathsf{eq}} = \mathsf{Th}(M^{\mathsf{eq}})$.

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Fact

For any theory T, T^{eq} eliminates imaginaries trivially: an element of a sort is the code for the corresponding equivalence class.

M^{eq}

Question

How much of M^{eq} do we need to add in order to eliminate imaginaries?

Balls in a valued field

Let K be a valued field, $a \in K$, $\gamma \in \Gamma$. Define

$B_{\geq\gamma}(a) = \{x \in K : v(x-a) \geq \gamma\}$the closed ball center a radius γ

$B_{>\gamma}(a) = \{x \in K : v(x-a) > \gamma\}$ the open ball center a radius γ

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Theorem

In a p-adically closed field, the closed and open balls suffice to code the equivalence classes in one variable (Scowcroft, unpublished, early 90's).

Conjecture

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The closed and open balls suffice to code all the definable sets.

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The conjecture is false

The sorted language for valued fields

Need to find some n-dimensional version of the balls. Notice:

- $B_{\geq\gamma}(0)$ is interdefinable with γ.
- $B_{\geq\gamma}(0)/B_{>\gamma}(0) \simeq k$.

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- $B_{\geq\gamma}(0)$ is interdefinable with γ.
- $B_{\geq\gamma}(0)/B_{>\gamma}(0) \simeq k$.
- $B_{\geq\gamma}(0)$ is an $\mathcal{O}_K$-module, freely generated by any element of value γ.
- $B_{>\gamma}(a) = a + B_{>\gamma}(0) = a + \mathcal{M}B_{\geq\gamma}(0)$, so is a coset of an $\mathcal{O}_K$-module.

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The elements of $\mathcal{S}_n$ are free $\mathcal{O}_K$-submodules of K^n on n generators.

T_n

The elements of T_n are of the form $a + s/\mathcal{M}s$, where $s \in S_n$, $a \in s$. That is, elements of T_n are cosets of $\mathcal{O}_K$-submodules of K^n (or *torsors*).

More formal definition of the sorts

Let $B_n(K)$ be the group of upper-triangular invertible $n \times n$ matrices over K (a definable subset of K^{n^2}). Let $B_n(\mathcal{O}_K)$ be the corresponding group of matrices which are invertible over $\mathcal{O}_K$.

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Let $B_{n,m}(\mathcal{O}_K)$ be the set of elements of $B_n(\mathcal{O}_K)$ such that, when reduced modulo the maximal ideal to the residue field, the mth column has a 1 on the diagonal, and 0's above.)

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$T_n = \bigcup_{m=1}^{n} B_n(K)/B_{n,m}(\mathcal{O}_K)$, the union over m of the set of left cosets of $B_{n,m}(\mathcal{O}_K)$ in $B_n(K)$.

The sorted language for valued fields

$S_n = B_n(K)/B_n(\mathcal{O}_K)$

Think of the columns of a matrix in $B_n(K)$ as the generators of an $\mathcal{O}_K$-module in K^n. Then two matrices are equivalent iff their columns generate the same $\mathcal{O}_K$-module. Thus the elements of $\mathcal{S}_n$ are $\mathcal{O}_K$-modules.

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$T_n = \bigcup_{m=1}^{n} B_n(K)/B_{n,m}(\mathcal{O}_K)$

Two matrices are equivalent modulo $B_{n,m}(\mathcal{O}_K)$ iff their columns generate the same $\mathcal{O}_K$-module and their mth columns determine the same coset of this $\mathcal{O}_K$-module modulo the maximal ideal. Thus elements of T_n are cosets of reductions of $\mathcal{O}_K$-modules modulo the maximal ideal (or *torsors*).

The geometric language

We define the *geometric* language for valued fields to include the following (possibly with additional structure):

- the field K with the field language
- the value group $\Gamma \cup \{\infty\}$ with the group language
- the residue field k with the field language
- the $\mathcal{O}_K$-modules $\mathcal{S}$
- the torsors $\mathcal{T}$

Theories of valued fields which eliminate imaginaries in the geometric language.

Theorem

The theory of algebraically closed valued fields has elimination of imaginaries in the geometric language (Haskell, Hrushovksi and Macpherson, 2001, to appear Crelle).

Theories of valued fields which eliminate imaginaries in the geometric language.

Theorem

The theory of p-adically closed fields has elimination of imaginaries in the geometric language of p-valued fields. (In fact, the sorts $\mathcal{T}$ are not needed as the valuation is discrete.) (Hrushovski and Martin, 2003)

Theories of valued fields which eliminate imaginaries in the geometric language.

Theorem

The theory of real closed valued fields has elimination of imaginaries in the geometric language of ordered fields (Mellor, PhD thesis 2003, to appear APAL).

A word on the proofs.

Proposition

Let T be a theory in a multisorted language with sorts $\{R_i\}$ and a dominant sort D. Suppose that for all models of T,

- *all definable subsets of D in one variable are coded, and*
- *for all definable functions f from D to one of the sorts, the graph of f is coded.*

Then T eliminates imaginaries.

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Proposition

In the above situation, in order to code the graph of f, it suffices to show that

$$\text{for all } a \in \mathrm{dom}(f),\ f(a) \in \mathrm{dcl}(a\mathrm{dcl}_{\mathrm{eq}}(f) \cap \bigcup R_i).$$

More recent results

A structural theorem: approximate statement

Let T be a theory of valued fields and suppose it has

- elimination of field quantifiers
- elimination of *linear imaginaries*
- orthogonality between the value group and the residue field

Then T eliminates imaginaries in the geometric language.
(Hrushovski, preprint 2004)

In particular, this extends the elimination of imaginaries results to Henselian valued fields.

Valued fields with analytic structure

Restricted power series

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It has been shown (Denef-van den Dries, 1988 for the first two, Lipshitz 1993 for ACVF) that the corresponding theory has quantifier elimination.

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The theory of real closed fields with restricted analytic functions is o-minimal (Gabrielov 1968).

The theory of p-adically closed fields with restricted analytic functions is P-minimal (van den Dries, Haskell, Macpherson 1999)

The theory of algebraically closed valued fields with separated power series is C-minimal (Lipshitz and Robinson, 1998).

Valued fields with analytic structure

Conjecture

Work in progress (Haskell and Lippel): The theory of ACVF with separated power series eliminates imaginaries in the geometric language.

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Further conjecture

Elimination of imaginaries for the real and p-adic theories with restricted analytic functions will follow in the same way as in the purely algebraic case.

Valued fields with a derivation

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A derivation on a field is a function from the field to itself which is a homomorphism on the additive structure and respects the Leibniz rule on multiplication.

Scanlon (2000) has shown quantifier elimination for the theory of differentially closed valued fields.

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Conjecture

Work in progress (Haskell, Scanlon and Yaffe): The theory of DCVF eliminates imaginaries in the geometric language.

Conclusions

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The role of multisorted structures

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Understanding valued fields

The geometric sorts described here — the $\mathcal{O}_K$-modules and their torsors — are key to understanding the existentially closed models of valued fields, and probably also their expansions.

Conclusions

Methods of proving elimination of imaginaries

The methods of proof provide new insight into the issues of eliminating imaginaries, and new applications of stability-theoretic methods in the analysis of non-stable theories.