Sample Test 3:

#1. $n = 8 < 30$ σ unknown

$\Rightarrow$ t test

$$\boxed{\begin{aligned} \bar{x} &= 11.47125 \\ s &= 0.02531939 \end{aligned}}$$

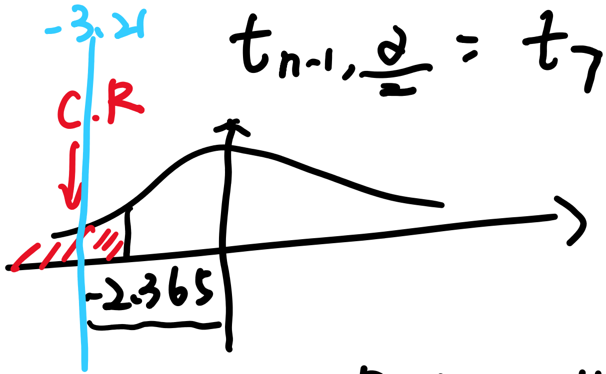
Hypothesis:
H0:
HA: $\neq$

Test Statistic:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{11.47125 - 11.5}{0.02531939/\sqrt{8}} = -3.211661$$

Critical value

$$t_{n-1, \frac{\alpha}{2}} = t_{7, \frac{5\%}{2}} = 2.365$$



Conclusion: Reject H0.

#2.

#3.

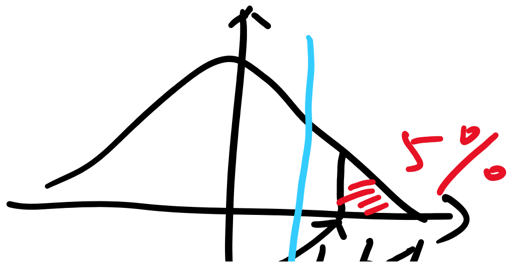
#4.

#5.

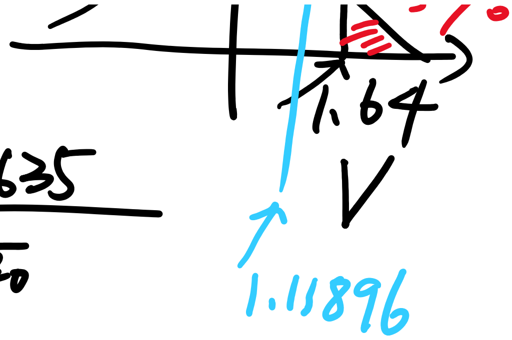
#6. $n = 40 > 30$

$\therefore$ Z test

$\begin{cases} H_0 \\ H_A: \mu > 0.635 \end{cases}$



$\{ H_A: \mu > 0.635$



Test statistic:

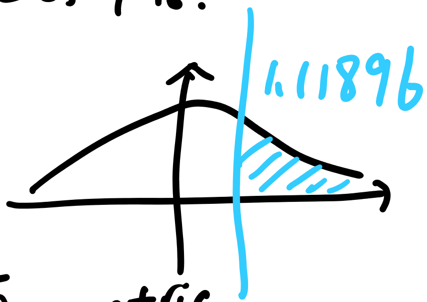
$$z = \frac{0.6373 - 0.635}{0.013/\sqrt{40}}$$

$$= 1.11896$$

$$Z_{5\%} = 1.64$$

$\therefore$ do not reject H_0.

#7 p-value.



$= P(Z \geq 1.12)$

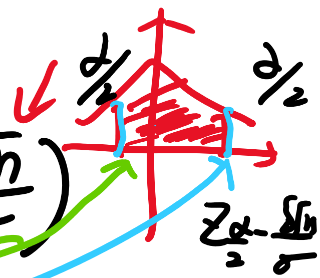
$= P(Z \leq -1.12)$ ← Symmetric.

$= 0.1314$

#8. $1-\beta$

$\beta = P(\text{accept } H_0 \mid H_A \text{ is True})$

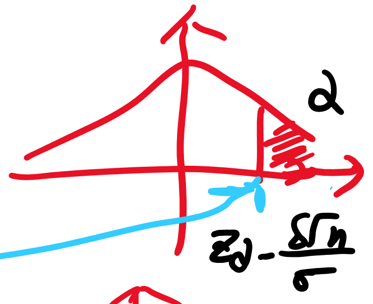
Two tail in the formula.



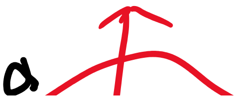
$$\beta = P\left(Z \leq z_{\frac{\alpha}{2}} - \frac{\delta\sqrt{n}}{\sigma}\right) - P\left(Z \leq -z_{\frac{\alpha}{2}} - \frac{\delta\sqrt{n}}{\sigma}\right)$$

One tail '>'

$$\Rightarrow \beta = P\left(Z \leq z_{\alpha} - \frac{\delta\sqrt{n}}{\sigma}\right) \quad ②$$



One tail '<'



One tail '<'

$\beta = 1 - P\left(Z \le -Z_\alpha - \frac{\delta\sqrt{n}}{\sigma}\right)$ ③

$-Z_\alpha - \frac{\delta\sqrt{n}}{\sigma}$

0.638 0.635

$\delta = \mu - \mu_0$
$= 0.003$

#8 β using 2nd equation

$= P\left(Z \le Z_\alpha - \frac{\delta\sqrt{n}}{\sigma}\right)$

$= P\left(Z \le Z_{5\%} - \frac{0.003\cdot\sqrt{40}}{0.013}\right)$

$= P\left(Z \le 1.64 - \frac{0.003\sqrt{40}}{0.013}\right) \Leftarrow$

$= P(Z \le 0.18)$

$= 0.5714$

1.645

power $1-\beta = 0.4286 \approx 0.426$

#9

$n = \sigma^2 \frac{(Z_{\frac{\alpha}{2}} + Z_\beta)^2}{\delta^2} \Leftarrow$ two tail

$n = \sigma^2 \frac{(Z_\alpha + Z_\beta)^2}{\delta^2} \Leftarrow$ one tail

$= \frac{0.013^2\,(\overset{Z_\alpha}{1.64} + \overset{Z_\beta}{1.28})^2}{0.003^2}$

$Z_{10\%} = 1.28$

$Z_{5\%} = 1.64$

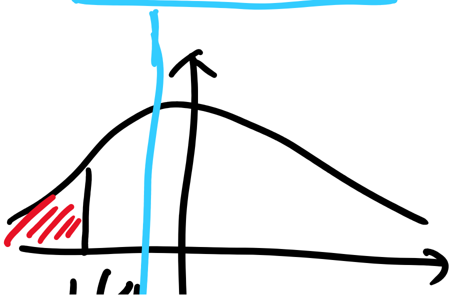
$= 160.6556$

#10

$Z = \frac{\frac{15}{1000} - 0.02}{\sqrt{\frac{0.02\cdot 0.98}{1000}}} = -1.129385$

HA: $p_0 < 0.02$.

$Z_\alpha = -1.64 = Z_{5\%}$



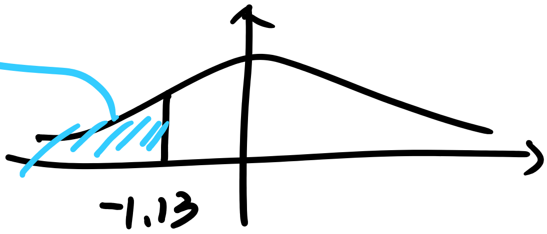
$z_\alpha = -1.64 = z_{5\%}$

$\therefore$ do not reject.

\# 11. p-value

$= P(Z \le -1.13)$

$= 0.1292$



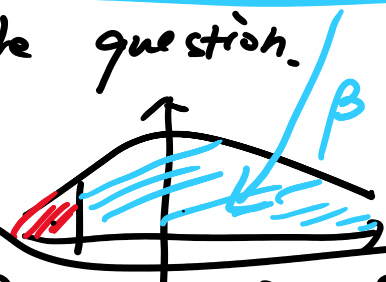
\# 12

one tail

$$\beta = 1 - P\left(Z \le \frac{p_0 - p - z_\alpha\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right)$$

$p = 0.01 \Leftarrow$ specified in the question.

$p_0 = 0.02 \Leftarrow$ Hypothesis.



Two tail

$$\beta = P\left(Z \le \frac{p_0 - p + z_{\frac{\alpha}{2}}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right) - P\left(Z \le \frac{p_0 - p - z_{\frac{\alpha}{2}}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right)$$

similar means β

$\beta = 1 - P(Z \le 0.87)$

$= 1 - 0.8078$

$= 0.1922$

\#13.

\#14

\#15

\#16 not required

\#17. $H_0: \mu_1 = \mu_2$

#17. $H_0: \mu_1 = \mu_2$

$H_A: \mu_1 < \mu_2$

unequal variance

Test Statistic

$$t = \frac{\overset{\mu_1}{0.8153} - \overset{\mu_2}{0.82524}}{\sqrt{\frac{0.0409^2}{12} + \frac{0.01794^2}{10}}}$$

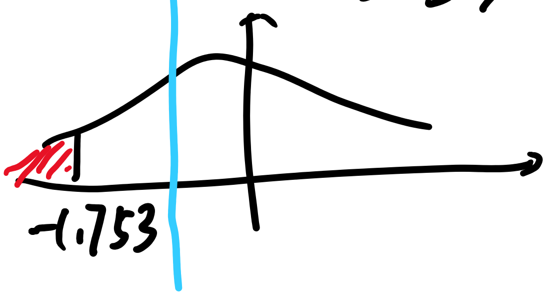
$\frac{s_1^2}{n_1}$ $\frac{s_2^2}{n_2}$

$= -0.7588335.$

$\nu = \underbrace{15.64628}_{\text{always round to the floor.}} = 15$

-0.76

$t_{15, 5\%} = 1.753$



$\therefore$ do not reject H_0.

#18 p-value

$t_{0.25,15} = 0.691 < t = 0.76 < t_{15,0.2} = 0.866$

$0.2 < p\text{-value} < 0.25$

#19 95% CI. two sided.

$(0.8153 - 0.82524) \pm 2.131 \cdot \sqrt{\frac{0.0409^2}{12} + \frac{0.01794^2}{10}}$

$= (-0.0378548, 0.0179748)$

#20 Test of β_1

$$t_{n-2} = \frac{b_1}{\sqrt{\hat{\sigma}^2 / S_{xx}}}$$

S_{xx}

$= \sum x_i^2 - n(\bar{x})^2$

$= 344634 - 342792$

$= 1842$

where $\hat{\sigma}^2 = \frac{SSE}{n-2} = MSE = 197.9$

$$t_{n-2} = \frac{0.297}{\sqrt{197.9/1842}}$$

$= 0.9061047$

#23.

$$SSE = \sum_{i=1}^{4} (n_i - 1) \cdot S_i^2$$

$= (38-1) 13.04^2 + (38-1) \cdot 12.12^2$

$+ (13-1)\, 9.71^2 + (11-1) \cdot 11.09^2$

$= 14087.92.$

$a = 4 \quad N = 100.$

grand mean

$$\bar{Y}_{..} = \frac{38 \cdot 135.16 + 38 \cdot 129.42 + 13 \cdot 125.23 + 11 \cdot 122.09}{100}$$

$= 130.2502$

$$SSTr = \sum_{i=1}^{a} n_i (\bar{Y}_{i.} - \bar{Y}_{..})^2$$

$38 \cdot (135.16 - 130.2502)^2$

$$
\begin{aligned}
&= 38 \cdot (135.16 - 130.2502)^2 \\
&+ 38 \cdot (129.42 - 130.2502)^2 \\
&+ 13 \cdot (125.23 - 130.2502)^2 \\
&+ 11 \cdot (122.09 - 130.2502)^2 \\
&= 2002.33
\end{aligned}
$$

$$MSTr = \frac{2002.33}{3} = 667.4443$$

$$MSE = \frac{14087.92}{96} = 146.7492.$$

$$F = \frac{MSTr}{MSE} = 4.548197.$$