Review 2

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Sample Test 1:

1. $\binom{3}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right) = 0.140625$.

2. A = only lower cases

B = exactly 1 lower case m.

$\# A = 26^6$

$\# B = \binom{6}{1} \cdot 35^5$

$\# A \cap B = \binom{6}{1} \cdot 25^5$

$p(A \cup B) = p(A) + p(B) - p(A \cap B)$

$\# \text{total} = 36^6$

$$p(A \cup B) = \frac{\# A + \# B - \# A \cap B}{\# \text{total}}$$

$= 0.2597656$

3. $p = \frac{26^3}{36^3}$ ← one password, no digit.

Geometric (p)

$E(X) = \frac{1}{p} = 2.654529$.

4. $\lambda = 5 \cdot 3 = 15$

$$p(X=1) = \frac{e^{-15}\, 15^1}{1!} = 4.58853 \times 10^{-6}.$$

5. $\lambda = \lambda t = 1$

5. $\lambda = 3 \cdot 2 = 6$

$$p(x \geq 1) = 1 - p(x=0) = 1 - \frac{e^{-6} 6^0}{0!} = 1 - e^{-6}$$

6. Hypergeometric: $N = 20$, $k = 5$ choose 6.

$$p(x=2) = \frac{\binom{5}{2}\binom{15}{4}}{\binom{20}{6}} = 0.3521672$$

7. Negative Binomial $(r = 4)$ $(p = 0.1)$

$$E(x) = \frac{r}{p} = 40.$$

$$\binom{x-1}{r-1}$$

8. $$1 - [\underbrace{0.1^4}_{4} + \underbrace{\binom{4}{1} 0.1^4 0.9}_{4} + \underbrace{\binom{5}{2} 0.1^4 0.9^2}_{6}]$$

$= 0.99873$

9. 1. Binomial, $p = \binom{12}{0} 0.99^{12} + \binom{12}{1} 0.99^{11} 0.01^1$

$= 0.9938255.$

2. Negative Binomial $r = 3$, p, $x = 7$

$$\therefore p(x=4) = \binom{6}{2} p^3 (1-p)^4 = 2.14012 \times 10^{-8}$$

10. $0.2 \cdot 0.9 + 0.8 \cdot 0.1 = 0.26.$

11. total #: $x + 10$.

$\Rightarrow$ Negative Binomial $(r = 10, p = 0.07)$

$$p(x=5) = \binom{5+10-1}{10-1} 0.07^5 \, 0.93^{10}$$

12. 1. $p = 0.99^{12} = 0.8863849$

2. Negative Binomial $(r=3,\ p\uparrow)$

$\therefore E(X) = \frac{r}{p} = 3.3845334$

13. Hypergeometric:

$E(X) = np = 2.27272$

14. $V(X) = 10 \cdot \frac{25}{110} \cdot \frac{85}{110} \cdot \frac{110-10}{110-1} = 1.611191.$

without replacement.

15. 1. Hypergeometric: $N=50,\ K=5,\ n=8$

p(no more 2 components)

$$= \frac{\binom{5}{0}\binom{45}{8} + \binom{5}{1}\binom{45}{7} + \binom{5}{2}\binom{45}{6}}{\binom{50}{8}}$$

2. Binomial $(p\uparrow)$

p(no more 2 Batches)

$$= \binom{7}{0} p^7 (1-p)^0 + \binom{7}{1} p^6 (1-p)^1 + \binom{7}{2} p^5 (1-p)^2$$

16. $p(E_2|E_1) = \frac{204}{207}$

17. $p = \frac{\overset{\#A}{207} + \overset{\#(E_2 \cap E_3')}{8} + 4 - \overset{\#(A \cap (E_2 \cap E_3'))}{4}}{370 \leftarrow \#\text{total}}$

18. —

18. ___

19. 5 - Balls 1

3 - Balls 2

1 - Ball 3.

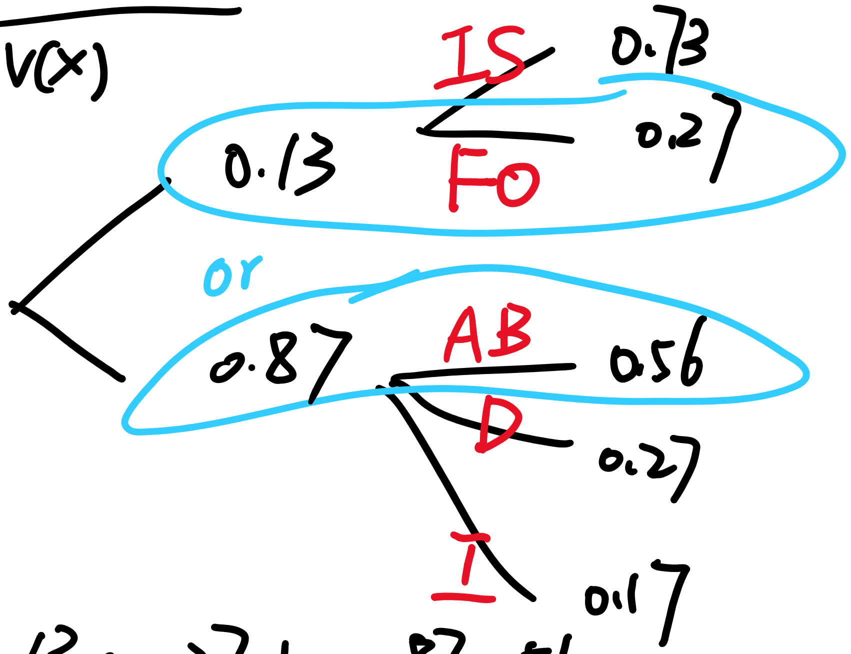
	$\frac{\binom{5}{2}}{\binom{9}{2}}$ $\frac{10}{36}$
∴ 1+1 = 2	$\binom{5}{1}\binom{3}{1}/\binom{9}{2}$ $\frac{15}{36}$
+ 1+2 = 3	$\binom{3}{2}+\binom{5}{1}\binom{1}{1}/\binom{9}{2}$ $\frac{8}{36}$
+ 2+2 = 4, 1+3 = 4	$\binom{3}{1}\binom{1}{1}/\binom{9}{2}$ $\frac{3}{36}$
+ 2+3 = 5	

$= E(X) = 2\cdot\frac{10}{36} + 3\cdot\frac{15}{36} + 4\cdot\frac{8}{36} + 5\cdot\frac{3}{36}$

20. ___

$V(X)$

21.



$0.13 \cdot 0.27 + 0.87 \cdot 0.56 = 0.5223$

22.

0.5 $\frac{\binom{1}{1}\binom{39}{2}}{\binom{40}{3}}$ ← Hypergeometric.

0.3 $\frac{\binom{2}{1}\binom{38}{2}}{\binom{40}{3}}$ $= 0.1206377.$

0.2 $\binom{3}{1}\binom{37}{2}$

0.2 $\dfrac{\binom{3}{1}\binom{37}{2}}{\binom{40}{3}}$

23. $f(x) = c\sin x, \quad 0 \le x \le \pi.$

$Var(x)$? $\Rightarrow \int f(x)\,dx \Rightarrow c = \frac{1}{2}$

$\therefore E(x) = \int x f(x)\,dx = \int \frac{1}{2} x \sin x \, dx$

$= \int -\frac{1}{2} x \, d\cos x$

I. By parts

$= \underline{-\frac{1}{2} x \cdot \cos x \Big|_0^{\pi} + \int \frac{1}{2} \cos x \, dx}$

$= -\frac{1}{2} x\cos x \Big|_0^{\pi} + \frac{1}{2} \sin x \Big|_0^{\pi}$

$= \frac{\pi}{2}$

$E(x^2) = \int x^2 f(x)\,dx = \frac{1}{2} \int x^2 \sin x \, dx$

I By parts twice

$= \underline{-\frac{1}{2} x^2 \cos x \Big|_0^{\pi} + \frac{1}{2} \int \cos x \cdot 2x \, dx}$

$= \frac{\pi^2}{2} + \underline{\sin x \cdot x \Big|_0^{\pi} - \int_0^{\pi} \sin x \, dx}$

$= \frac{\pi^2}{2} - 2$

$V(x) = E(x^2) - [E(x)]^2 = \frac{\pi^2}{4} - 2 = 0.4674011$

24. CDF $\rightarrow$ $f(x) = c\sin x, \quad 0 \le x \le \pi.$

$$F(x) = \int_0^x f(x)\,dx$$

$$= \int_0^x \frac{1}{2}\sin x\,dx$$

$$= \frac{1}{2}(1-\cos x)\,dx.$$

25.—27.

$$f(x) = \frac{c\,e^{-2(x-4)}}{1+e^{-2(x-4)}}, \quad x \geq 4.$$

$$\therefore \int f(x)\,dx = 1.$$

$$\int \frac{c\,e^{-2(x-4)}}{1+e^{-2(x-4)}}\,dx = 1.$$

→u

→u

$$\Rightarrow c \cdot \ln\left[1+e^{-2(x-4)}\right]\Big|_4^{\infty} \cdot \left(-\frac{1}{2}\right) = 1$$

$$\Rightarrow c = \frac{2}{\ln 2}$$

$$p = \int_5^{\infty} f(x)\,dx = \frac{\ln(1+e^{-2(x-4)})}{\ln 2}\Big|_{x=5} = 0.183184$$

→5 for each

$E(x) = np = 5.495552$ ⇐ Binomial. $n=30$. p

$V(x) = np(1-p) = 4.487582$

CDF.

$$\Rightarrow \frac{\ln(1+e^{-2(x-4)})}{\ln 2} = 0.2.$$

$$\Rightarrow x = 4.149809$$

Sample Test 2.

1. $p(X \le 1) = 1 - e^{-\frac{1}{6}\cdot 1}$

2. $1 - e^{-\frac{1}{6}x} = 0.9$

3. $\left[e^{-\frac{7}{6}}\right]^{10}$

4. $p(100 \le X \le 150)$

$$= p\left(\frac{100-0.5-500p}{\sqrt{500p(1-p)}} \le Z \le \frac{150+0.5-500p}{\sqrt{500p(1-p)}}\right)$$

where $p = e^{-\frac{7}{6}} = 0.3114$

$$= p(-5.43 \le Z \le -0.50)$$

$$= 0.3085$$

5. $p(5.1 \le X \le 5.3)$

$$= p\left(\frac{5.1-5}{0.2} \le Z \le \frac{5.3-5}{0.2}\right)$$

$$= p(0.5 \le Z \le 1.5)$$

$$= 0.9332 - 0.6915$$

$$= 0.2417$$

6. $p = 0.2417$ from Q5

6. $p = 0.2417$ from Q5

$\therefore E(x) = np = 400 \cdot 0.2417 = 96.68$

$V(x) = \sigma^2 = npq = 400 \cdot 0.2417 \cdot (1 - 0.2417)$

$\therefore p(x \geq 100) = p(Z \geq \frac{99.5 - \mu}{\sigma})$

7. $p(Z > \frac{x-5}{0.2}) = 0.92$

$Z_{0.08} = -1.41 \quad \therefore \frac{x-5}{0.2} = -1.41$

$\Rightarrow x = 4.718$

8. $p(x \leq 4.8) = 0.1$

$\therefore p(Z \leq \frac{4.8-5}{\sigma}) = 0.1$

$\therefore Z = -1.28 \quad \therefore -1.28 = \frac{4.8-5}{\sigma}$

$\therefore \sigma = 0.15625$

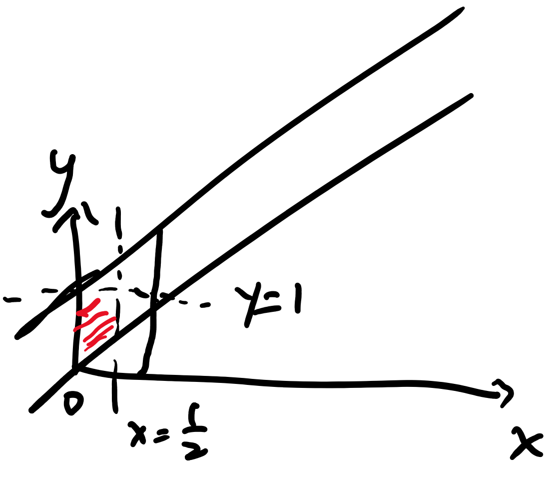
9. $f(x,y) = cx(1+y), \quad 0 \leq x \leq 1, \quad 0 < y - x < 1$

find c.

$\therefore \int_0^1 \int_x^{x+1} cx(1+y)\, dy\, dx$

$\Rightarrow \therefore c = \frac{12}{13}$

10. $f(x,y) = \frac{12}{13} x(1+y)$

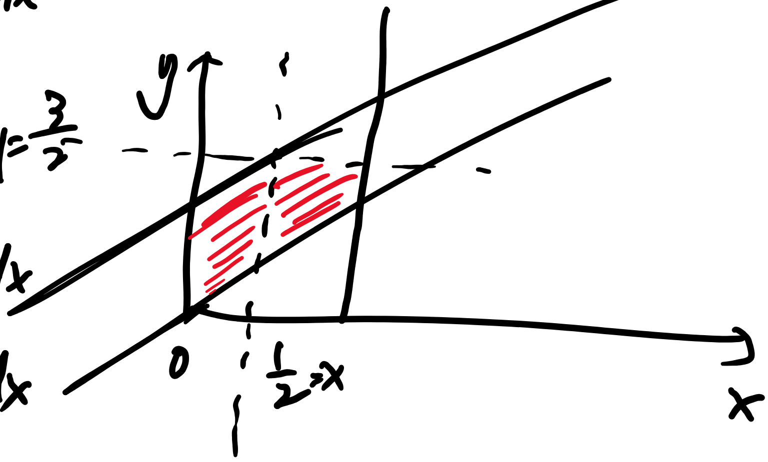


$\therefore p(0 \leq x \leq \frac{1}{2}, 0 < Y < 1)$

$= \int_0^{\frac{1}{2}} \int_x^1 \frac{12}{13}(x + xy)\, dy\, dx$

$= \int_0^{\frac{1}{2}} \int_x^1 \frac{12}{13}(x + xy)\, dy\, dx$

11. $P(Y \le \frac{3}{2})$ $\quad Y = \frac{3}{2}$



$= \left[\int_0^{\frac{1}{2}} \int_x^{x+1} f(x,y)\, dy\, dx + \int_{\frac{1}{2}}^1 \int_x^1 f(x,y)\, dy\, dx \right.$

12. $f(x) = \int_x^{x+1} f(x,y)\, dy\, dx$

$= \frac{6}{13} x(2x+3)$

13. $E(xy) = \int_0^1 \int_x^{x+1} \frac{12}{13} xy \cdot x(1+y)\, dy\, dx$

$E(x) = \int_0^1 \int_x^{x+1} \frac{12}{13} x\, x(1+y)\, dy\, dx$

$E(y) = \int_0^1 \int_x^{x+1} \frac{12}{13} y\, x(1+y)\, dy\, dx$

$\mathrm{cov}(X,Y) = E(XY) - E(X)E(Y)$

14. $Y = \sum X$

$\therefore E(Y) = 12.1 \cdot 24$

$V(Y) = 24^2 \cdot 1.1$

$\therefore P(Y > 300) = P\left(Z > \frac{300 - 24 \cdot 12.1}{\sqrt{24^2 \cdot 1.1}} \right)$

$\therefore p(T > 300) = p\left(Z > \dfrac{\quad}{\sqrt{24^2 \cdot 1.1}}\right)$

$= p(Z > 1.78)$

$= 0.0375.$

15. $p(12 \le X \le 12.2)$

$= p\left(\dfrac{-0.1}{1.1/\sqrt{n}} \le Z \le \dfrac{0.1}{1.1/\sqrt{n}}\right)$

$= 0.96$

$\therefore Z_{0.98} = 2.05$

$\therefore \dfrac{0.1}{1.1/\sqrt{n}} = 2.05$

$\therefore n = 508.5025$

16. $468 + 654 = 1122$

17. $\dfrac{11 \cdot 15.3 - 12.2}{10}$

18 $Q_1 = 77 \cdot \frac{3}{4} + 84 \cdot \frac{1}{4} = 78.75$

$Q_3 = 97 \cdot \frac{1}{4} + 101 \cdot \frac{3}{4} = 100$

25 34 77 84 87 88 91 93 97 101 104 130

$n = 12$ $\begin{cases} \dfrac{n+1}{4} = \dfrac{13}{4} \text{th} \\ \dfrac{3(n+1)}{4} = \dfrac{39}{4} \text{th} \end{cases}$

19. $f(x) = \dfrac{x}{8}, \quad 3 < x < 5$

19. $f(x) = \frac{x}{8}$, $3 < x < 5$

$E(x) = \int_3^5 \frac{x}{8} \cdot x \, dx = \frac{98}{24}$

$E(x^2) = \int_3^5 \frac{x}{8} \cdot x^2 \, dx = \frac{x^4}{32}\Big|_3^5 = \frac{544}{32}$

$V(x) = E(x^2) - [E(x)]^2 = 0.3263888$

$\therefore \sigma_x = 0.5713$

$\therefore p(x \le 4.2)$

$= p\left(z \le \frac{4.2 - \frac{98}{24}}{0.5713/\sqrt{49}}\right)$

$= p(z \le 1.43)$

$= 0.9236$

20. $p\left(z \ge \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}\right)$ where exp(6) $\begin{cases} \mu = 6 \\ \sigma = 6 \end{cases}$

$= p\left(z \ge \frac{6.2 - 6}{6/\sqrt{32}}\right)$

$= p(z \ge 0.19)$

$= 0.4247$

24. $n = \frac{z_{0.995}^2}{E^2} \cdot 0.25$

$= 848.26$

25. $\underbrace{t_{14, 0.99}}_{\text{two tail}} = 2.977$ $\underbrace{t_{14, 0.95}}_{\text{two tail}} = 2.145$

$\therefore \begin{cases} ME = \frac{90.6781 - 83.9219}{2} = 3.3781 \\ s \end{cases}$

$\therefore$ 95% $\begin{cases} ME = \text{———} = [illegible] \\ ME = t_{14,\ 0.95} \cdot \frac{s}{\sqrt{n}} \end{cases}$

$\therefore \frac{s}{\sqrt{n}} = 1.57487$

$\therefore$ 99% $ME = 2.977 \cdot \frac{s}{\sqrt{n}} = 4.6883933$

$\therefore 87.3 \pm 4.6883933$

26. $\hat{p} = \frac{0.095970 + 0.304030}{2} = 0.2$

$\therefore ME = 0.104030$

$= 1.645 \sqrt{\frac{0.2 \cdot 0.8}{n}}$

27.

$\Rightarrow \bar{x} = 28.1$

$\therefore 1.4175 = Z \cdot \frac{5.2535}{\sqrt{45}}$

$\therefore Z = 1.81$

Note: ① There are two types in the questions

1. $Z = \frac{\hat{p} - p_0}{\sqrt{\ldots}}$

1. $Z = \frac{p - p_0}{\sqrt{\frac{p_0 \cdot p_0}{n}}}$

2. $\beta = 1 - P\left(Z \le -Z_\alpha - \frac{\delta\sqrt{n}}{\sigma}\right)$

(2) Some formulas, need to be memorized.

1. One tail sample size given α and β

$$n = \frac{(Z_\alpha + Z_\beta)^2}{\delta^2} \cdot \sigma^2.$$

2. ANOVA TABLE (two)

3. two sample tests for mean (from C.I.)

Variance Equal: $t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

Unequal: $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$