Stats 3Y03/3J04

Sample Test Questions for Test #1

Name:_____
(Last Name) (First Name)

Student Number:_____ **Tutorial Number:**_____

This test consists of 27 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. All questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Only the McMaster standard calculator Casio fx-991 is allowed.

1. Suppose that a disease is inherited via an **autosomal recessive** mode of inheritance. The implications of this mode of inheritance are that the children in a family each (independently) have a probability of $\frac{1}{4}$ of inheriting the disease. In a family of three children, find the probability that exactly 2 of the children inherit the disease.

(a) .0469 **(b)** .1406 **(c)** .2317 **(d)** .4635 **(e)** .0156

2. A computer system uses passwords constructed from the 26 letters (a-z, lowercase only) or 10 integers (0-9). If a 6 character password is randomly selected, find the probability that it consists only of lower case letters or contains exactly 1 lower case m.

(a) .2598 **(b)** .2867 **(c)** .1267 **(d)** .3412 **(e)** .1361

3. A computer system uses passwords constructed from the 26 letters (a-z, lowercase only) or 10 integers (0-9). Suppose that we continue to randomly generate three-character passwords until we obtain one that does not have any digits. What is the mean number of passwords that have to be generated in order to obtain one that does not have any digits?

(a) 3.8124 **(b)** 5.3615 **(c)** 1.6257 **(d)** 2.6545 **(e)** .3767

4. The number of people who enter a particular hospital emergency room in a given hour is a Poisson random variable with mean 3. Find the probability that in the next five hours exactly 1 person enters the emergency room.

(a) .00215 **(b)** .7468 **(c)** .1494 **(d)** 9.08×10^{-5} **(e)** 4.58853×10^{-6}

5. The number of messages sent to a computer bulletin board is a Poisson random variable with a mean of 2 messages per hour. Find the probability that at least 1 message is sent in the next 3 hours.

(a) .01487 **(b)** .004958 **(c)** .98513 **(d)** .9975 **(e)** .86247

6. A steel plate contains 20 bolts. Assume that 5 of those bolts are not torqued to the proper limit. Six bolts are selected at random, without replacement, and checked for torque. What is the probility that exactly 2 of the selected bolts are not torqued to the proper limit?

(a) .2966 **(b)** .3522 **(c)** .0135 **(d)** .0198 **(e)** .000258

7. In a clinical study, volunteers are tested for a gene that has been found to increase the risk for a disease. The probability that a person carries the gene (independently of other people) is 0.1. What is the mean number of people that have to be tested in order to find 4 people who carry the gene?

(a) 40 **(b)** 0.4 **(c)** 25 **(d)** 8 **(e)** 16

8. In a clinical study, volunteers are tested for a gene that has been found to increase the risk for a disease. The probability that a person carries the gene (independently of other people) is 0.1. What is the probability that we will have to test at least 7 people in order to find 4 people who carry the gene?

(a) .9982 **(b)** .00255 **(c)** .9987 **(d)** 1.458×10^{-6} **(e)** .00346

9. An automated egg carton loader has a 1% probability of cracking an egg (for each egg loaded). Suppose that you continue to inspect cartons (containing one dozen eggs) until you find 3 cartons that have fewer than 2 cracked eggs. Find the probabiliy that you will have to inspect 7 cartons in total.

(a) .00129 **(b)** 1.4409×10^{-5} **(c)** .000326 **(d)** 2.14013×10^{-8} **(e)** .01854

10. An assembly consists of two mechanical components. Suppose that the probabilities that the first and second components meet specifications are 0.8 and 0.9, respectively. Assume that the components are independent. Let X be the number of components in the assembly that meet specifications. Find $P(X = 1)$

(a) 0.26 **(b)** 0.3 **(c)** .28 **(d)** .98 **(e)** .37

11. Computer chips from a certain supplier have a 7% chance of containing a manufacturing defect. Suppose that we continue to test chips from this manufacturer until we find 10 that have no defects. Let X be the number of defective chips that we have found at the time of the 10th good chip. Find $P(X = 5)$.

(a) 1.4732×10^{-4} **(b)** .001957 **(c)** .0016285 **(d)** 1.3742×10^{-16} **(e)** .0023761

12. An automated egg carton loader has a 1% probability of cracking an egg (for each egg loaded). On average, how many cartons (containing one dozen eggs) would you have to inspect in order to find 3 cartons with no cracked eggs?

(a) 2.3867 **(b)** 3.3845 **(c)** 10.622 **(d)** 5.829 **(e)** 6.2914

13. Suppose that a batch of electrical components contains 85 conforming items and 25 nonconforming items. A sample of 10 components are seleced from a batch. Find the mean of the number of nonconforming items in the sample.

(a) 1.2941 **(b)** 3.111111 **(c)** 2.27272 **(d)** 7.7273 **(e)** 1.797979

14. Suppose that a batch of electrical components contains 85 conforming items and 25 nonconforming items. A sample of 10 components are seleced from a batch. Find the variance of the number of nonconforming items in the sample.

(a) 1.75621 **(b)** 2.38124 **(c)** 1.18576 **(d)** 3.98152 **(e)** 1.61119

15. Electrical components from a supplier are shipped in batches of 50. 8 components are selected from each batch for inspection. If more than 2 components in the sample of 8 are found to be nonconforming then the batch is returned to the supplier. Suppose that each batch actually contains 45 conforming items and 5 nonconforming items. Find the probability that in the next 7 batches no more than 2 batches are returned to the supplier.

(a) .6439 **(b)** .8743 **(c)** .9887 **(d)** .9995 **(e)** .8908

16. Shafts are classified in terms of the machine tool that was used for manufacturing the shaft and conformance to surface finish and roundness.

Tool 1		**Roundness Conforms**	
		yes	no
Surface Finish Conforms	yes	200	1
	no	4	2

Tool 2		**Roundness Conforms**	
		yes	no
Surface Finish Conforms	yes	145	4
	no	8	6

A shaft is selected at random. Let E_1 be the event that Tool 1 was used, let E_2 be the event that it conforms to roundness specifications. Find $P(E_2 \mid E_1)$.

(a) .9649 **(b)** .9855 **(c)** .5714 **(d)** .5514 **(e)** .5268

17. Shafts are classified in terms of the machine tool that was used for manufacturing the shaft and conformance to surface finish and roundness.

Tool 1		**Roundness Conforms**	
		yes	no
Surface Finish Conforms	yes	200	1
	no	4	2

Tool 2		**Roundness Conforms**	
		yes	no
Surface Finish Conforms	yes	145	4
	no	8	6

A shaft is selected at random. Let E_1 be the event that Tool 1 was used, let E_2 be the event that it conforms to roundness specifications, and let E_3 be the event that it conforms to surface finish specifications. Find $P[E_1 \cup (E_2 \cap E_3')]$.

(a) .5811 **(b)** .8327 **(c)** .6415 **(d)** .5228 **(e)** .4751

18. A box contains 3 balls, labeled "1", "2", and "3". Two balls are randomly selected *with* replacement (i.e., the first ball is put back into the box before the second ball is selected). Let X be the total of the values on the two balls selected. Find $P(X \geq 4)$.

(a) $\frac{2}{3}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{5}{9}$ **(d)** $\frac{4}{9}$ **(e)** $\frac{2}{9}$

19. A box contains 5 Balls labeled with the number "1", 3 balls labeled with the number "2", and 1 ball labeled with the number "3". Two balls are selected, *without* replacement. Let X be the total of the values on the two balls. Find the mean of X.

(a) 3.27778 **(b)** 3.56671 **(c)** 2.87512 **(d)** 3.11111 **(e)** 3.84316

20. A box contains 5 Balls labeled with the number "1", 3 balls labeled with the number "2", and 1 ball labeled with the number "3". Two balls are selected, *without* replacement. Let X be the total of the values on the two balls. Find the variance of X.

(a) .820988 **(b)** .731671 **(c)** 1.287214 **(d)** 1.435871 **(e)** 1.552372

21. Heart failures are due to either natural occurrences (87%) or outside factors (13%). Outside factors are related to induced substances (73%) or foreign objects (27%). Natural occurrences are caused by arterial blockage (56%), disease (27%), and infection (17%). Determine the probability that a heart failure is due to foreign objects or arterial blockage.

(a) .5052 **(b)** .5147 **(c)** .5223 **(d)** .4879 **(e)** .4752

22. Components of a certain type are shipped to a supplier in batches of 40. Suppose that 50% of all such batches contain one defective component, 30% contain two defective components, and 20% contain three defective components. If you select 3 components at random (and without replacement) from a batch, find the probability that you will get exactly 1 defective component.

(a) .110340 **(b)** .032561 **(c)** .120638 **(d)** .158574 **(e)** .241390

23. Suppose that a random variable has probabilty density function $f(x) = c \sin x$, $0 \leq x \leq \pi$. Find the variance of X.

(a) .467401 **(b)** .589746 **(c)** .304629 **(d)** .254791 **(e)** .295862

24. Suppose that a random variable has probabilty density function $f(x) = c \sin x$, $0 \le x \le \pi$. Find the cumulative distribution function of X.

(a) $F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{3}(1 - \cos x) & 0 \le x \le \pi \\ 1 & x > \pi \end{cases}$ **(b)** $F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{3}\cos x & 0 \le x \le \pi \\ 1 & x > \pi \end{cases}$

(c) $F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2}\cos x & 0 \le x \le \pi \\ 1 & x > \pi \end{cases}$ **(d)** $F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2}(1 - \cos x) & 0 \le x \le \pi \\ 1 & x > \pi \end{cases}$

(e) $F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2}\cos x & 0 \le x \le \pi \\ 0 & x > \pi \end{cases}$

25. Suppose that the diameter of a hole (in millimeters) drilled in a sheet metal component has probability density function

$$f(x) = \frac{ce^{-2(x-4)}}{1 + e^{-2(x-4)}},\ x \ge 4$$

If we drill 30 such holes, find the mean of the number of holes that have a diameter greater than 5 millimeters.

(a) 5.49355 **(b)** 5.77921 **(c)** 4.97956 **(d)** 4.88047 **(e)** 4.78782

26. Suppose that the diameter of a hole (in millimeters) drilled in a sheet metal component has probability density function

$$f(x) = \frac{ce^{-2(x-4)}}{1 + e^{-2(x-4)}},\ x \ge 4$$

If we drill 30 such holes, find the variance of the number of holes that have a diameter greater than 5 millimeters.

(a) 3.23751 **(b)** 2.66448 **(c)** 3.78543 **(d)** 4.19892 **(e)** 4.48758

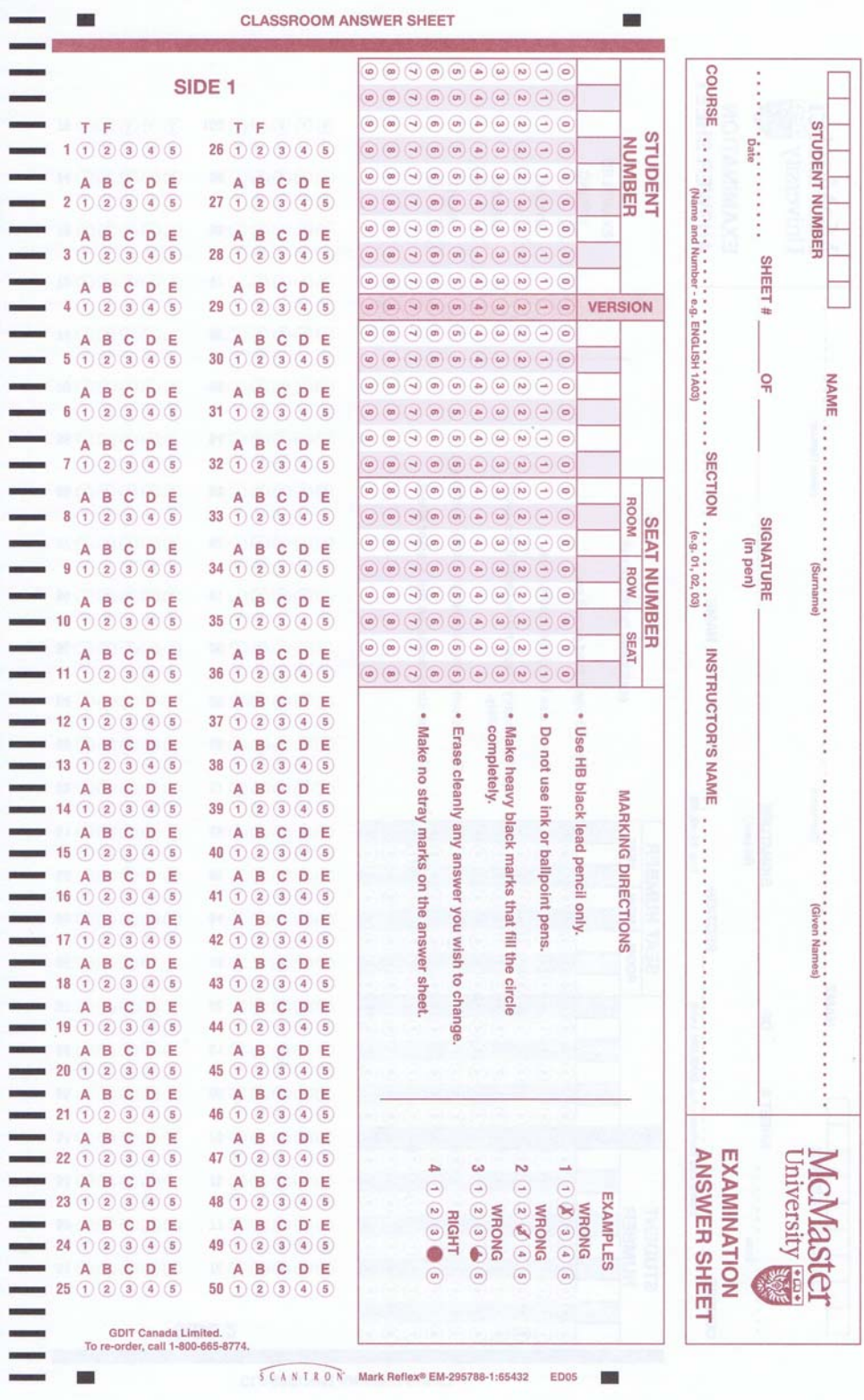
27. Suppose that the diameter of a hole (in millimeters) drilled in a sheet metal component has probability density function

$$f(x) = \frac{ce^{-2(x-4)}}{1 + e^{-2(x-4)}},\ x \ge 4$$

Fill in the blank. 80% of the holes drilled have a diameter greater than _____ mm.

(a) 6.6367 **(b)** 7.1365 **(c)** 4.1498 **(d)** 5.8732 **(e)** 4.3509

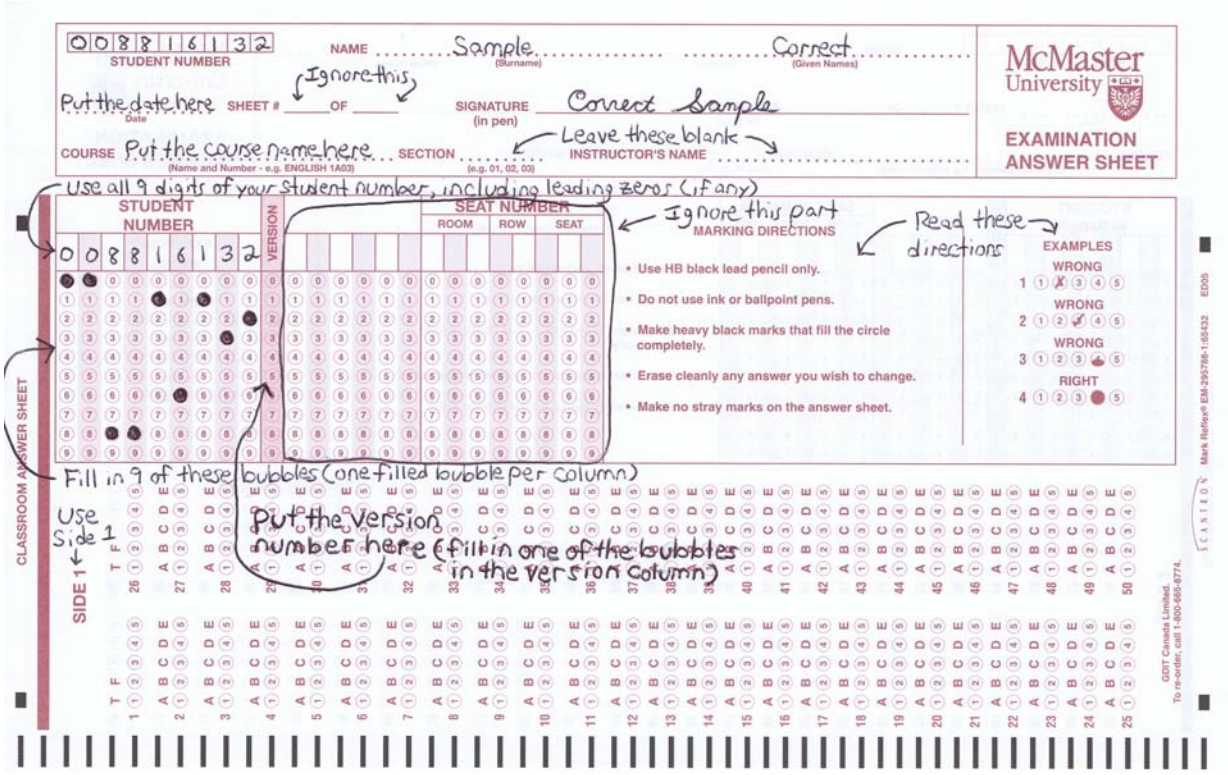
28. Correctly fill out the bubbles corresponding to all 9 digits of your student number, as well as the version number of your test in the correct places on the computer card. (Use the below computer card for this sample test.)



Answers

1. b **2.** a **3.** d **4.** e **5.** d **6.** b **7.** a **8.** c **9.** d **10.** a
11. c **12.** b **13.** c **14.** e **15.** d **16.** b **17.** a **18.** a **19.** d **20.** a
21. c **22.** c **23.** a **24.** d **25.** a **26.** e **27.** c

28.



NOTE: On the sample tests, a version number is not given. On the actual tests, it will say "Version X" at the top, where X is the version number that you will have to fill in on the computer card. The sample above assumes that your student number is 008816132. On the actual test, you will have to fill in the bubbles corresponding to YOUR student number (not 008816132).