Stats3Y03/3J04 Test 2 (Version 1)

Instructor: Mu He **Time: 7:00 - 8:00 P.M. May 28th, 2018**

First Name:____

Last Name:____

Student ID:____

There are total 16 multiple choice questions for this test. Each question carries equal marks. All questions must be answered on the COMPUTER CARD with an HB PENCIL. You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Only the McMaster standard calculator Casio FX-991 MS or MS Plus is allowed.

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1. Let $f(x, y) = 6x, \quad x > 0, \quad y > 0, \quad x + y < 1$. Find $P(X < \frac{1}{4})$

(a) 0.3827 (b) 0.2593 (c) 0.2126 (d) 0.1875 (e) 0.1563

2. Suppose that the random variable X has the following distribution.

$$f(x) = 3x^2, \quad 0 < x < 1$$

Suppose that a random sample of $n = 2$ is selected from this distribution. Find the variance of $\bar{X}$.

(a) $\frac{3}{80}$ (b) $\frac{1}{80}$ (c) $\frac{3}{160}$ (d) $\frac{1}{40}$ (e) $\frac{3}{40}$

3. Supose that the number of customers coming per day for the Shoppers is 30, following a Poisson Distribution. What is the probability that for the next month, there will be 25 to 35 inclusive customers.

(a)0.6914 (b) 0.6802 (c) 0.6826 (d) 0.7050 (e) 0.6735

4. The life of CPU is exponentially distributed with a mean lifetime of six years. If you have owned your CPU for four years, what is the probability that it will fail within the next two years.

(a)0.6321 (b) 0.2835 (c) 0.7364 (d) 0.9999 (e) 0.3015

5. The CPU of a personal computer has a lifetime that is exponential distributed with a mean of six years. If you buy 3 CPUs, find the probability that they all fail within nine years.

(a) 0.011109 (b) 0.2231302 (c) 0.7768698 (d) 0.4688617 (e) 0.408722

6. Let $f(x, y) = x + y$, $0 < x < 1$, $0 < y < 1$. Find $E(XY)$.

(a)$\frac{3}{2}$ (b) 1 (c) $\frac{1}{4}$ (d) $\frac{1}{3}$ (e) $\frac{1}{2}$

7. A data set with $n = 11$ observations has a sample mean of $\bar{x} = 15$. Suppose that one of the values in the data set 12 is now removed from the data set. What is the value of $\bar{x}$ for this new sample?

(a) 15.3 (b) 15.1 (c) 14.9 (d) 15 (e) 15.2

8. Suspension helmets used by motorcyle riders and automobile race-car drivers are subjected to an impact test in order to estimate the percentage of such helmets that are damaged. What is the minimum sample size required if we wish to be 99% confident that the error in estimating this percentage is less than 0.02?

(a) 4161 (b) 3383 (c) 2944 (d) 2637 (e) 3001

9. The solar energy consumed (in trillion BTU) during 4 selected years in the United states, has a mean $\bar{x} = 15$ and a sample variance $s^2 = 5$. Find a 95% confidence interval for the mean yearly solar energy consumed.

(a) (7.045, 22.955) (b) (11.4424, 18.5576) (c) (12.36886, 17.63114) (d) (13.161, 16.839)
(e) (12.80869 17.19131)

10. Which of the following Stem and leaf plot is correct? $N = 23$

(a) Stem — Leaf

3	1	1 2 2
7	2	5 6 6 8
(4)	3	0 3 3 4
12	4	7 8
10	5	7 9
8	6	1 2 3
5	7	3 4
3	8	4 1 3

(b) Stem — Leaf

3	1	1 2 2
7	2	5 6 6 8
10	3	0 3 3 4
(2)	4	7 8
10	5	7 9
8	6	1 2 3
5	7	3 4
3	8	4 1 3

(c) Stem — Leaf

3	1	1 2 2
7	2	5 6 6 8
11	3	0 3 3 4
(2)	4	7 8
10	5	7 9
8	6	1 2 3
5	7	3 4
3	8	4 1 3

(d) Stem — Leaf

3	1	1 2 2
4	2	5 6 6 8
4	3	0 3 3 4
(2)	4	7 8
2	5	7 9
3	6	1 2 3
2	7	3 4
3	8	4 1 3

(e) Stem — Leaf

3	1	1 2 2
7	2	5 6 6 8
11	3	0 3 3 4
13	4	7 8
(2)	5	7 9
8	6	1 2 3
5	7	3 4
3	8	4 1 3

11. Suppose that, we have a data set: 1, 2, 27, 6, 7, 9, 12, 15, 18, 19, 5. What is the IQR for this data set?

(a) 15 (b) 13 (c) 12 (d) 14 (e) 11

12. Adobe bricks for construction have a mean weight of 12.1 pounds, with standard deviation 1.1 pounds. Assume that the weights of adobe bricks are independent normal random variables. The company wants to ship the bricks in packages and guarantee that the average brick weight in a package is between 12.0 and 12.2 pounds. How many bricks should be put in each package so that 96% of all such packages will meet the guarantee?

(a) 1024 (b) 623 (c) 487 (d) 821 (e) 509

13. A batch of electrical components are tested and 85% of them lasted longer than 1000 hours. Based on this data a confidence interval was then produced to estimate the proportion of such electrical components that last longer than 1000 hours. One of the below intervals is the confidence interval that was produced. Which one?

(a) (0.837,0.867) (b) (0.838, 0.868) (c) (0.808,0.876) (d) (0.834, 0.860) (e) (0.820,0.880)

14. Consider the following data set $n = 6$,

$$12.5, 15.7, 18.3, 25.1, 31.2, 47.5$$

If we were to construct a normal probability plot for this data set, which of the following would be one of the points on the plot?

(a)(0.67, 15.7) (b) (a)(-0.67, 31.2) (c) (-1.38,12.5) (d) (-0.97, 12.5) (e) (-0.21,15.7)

15. Let $X_1, X_2, ..., X_7$ denote a random sample from a population having mean μ and variance σ^2. Which of the follwoing are the unbiased estimator of μ?

(i) $\hat{\theta_1} = \frac{2X_1+X_3+4X_6}{7}$ (ii)$\hat{\theta_2} = \frac{2X_1-X_2+2X_7}{4}$ (iii) $\hat{\theta_3} = \frac{2X_3+3X_6-2X_1}{3}$ (iv) $\hat{\theta_4} = \frac{4X_3+3X_6-2X_1}{5}$

(a) all of them (b) (i),(iii),(iv) (c) none of them (d) (i),(ii),(iv) (e) (ii),(iii),(iv)

16. The answer for this question is (a), if you could bubble both this question's answer and the version number right, you could get the mark for this question.

Answer: E C C B D D A A B C B E E C B A

Test 2 Formula Sheet

Continuous R.V.:

Mean (Expected Value): $\mathbb{E}(X) = \mu = \int_{-\infty}^{\infty} xf(x)dx$

Variance: $\mathbb{V}(X) = \sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$

C.D.F: $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)dt$

Common Distributions:

Normal Distribution (μ, σ^2):

$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$, $-\infty < x < \infty$

$\mathbb{E}(X) = \mu$, $\mathbb{V}(X) = \sigma^2$

Exponential Distribution (λ):

$f(x) = \lambda e^{-\lambda x}$, $x \geq 0$

$F(x) = P(X \leq x) = 1 - e^{-\lambda x}$

$\mathbb{E}(X) = \frac{1}{\lambda}$, $\mathbb{V}(X) = \frac{1}{\lambda^2}$

Normal Approximation to Binomial Distribution:

$$P(X \leq x) = P(X \leq x + 0.5) \approx P(Z \leq \frac{x + 0.5 - np}{\sqrt{np(1-p)}})$$

$$P(X \geq x) = P(X \geq x - 0.5) \approx P(Z \geq \frac{x - 0.5 - np}{\sqrt{np(1-p)}})$$

Normal Approximation to Poisson Distribution:

$$P(X \leq x) = P(X \leq x + 0.5) \approx P(Z \leq \frac{x + 0.5 - \lambda}{\sqrt{\lambda}})$$

$$P(X \geq x) = P(X \geq x - 0.5) \approx P(Z \geq \frac{x - 0.5 - \lambda}{\sqrt{\lambda}})$$

Marginal Distributions: $f_X(x) = \int f_{XY}(x,y)dy$, $f_Y(y) = \int f_{XY}(x,y)dx$

Correlation: $\rho_{XY} = \frac{Cov(X,Y)}{\sqrt{\mathbb{V}(X)\mathbb{V}(Y)}} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$

Standizing: $Z = \frac{X-\mu}{\sigma}$

Sample Mean: $\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$

Sample Variance: $s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} = \frac{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}{n-1}$

Q_1: The (n+1)/4 th number in the data set.

Q_3: The 3(n+1)/4 th number in the data set.

Outliers: $Q_1 - 1.5IQR, Q_3 + 1.5IQR$

Normal Probability Plot: $\Phi(z_j) = \frac{j-0.5}{n}$, $j = 1, 2, ..., n$

Central Limit Theorem Formula: $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

z-Confidence Interval for the Mean: $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

t-Confidence Interval for the Mean: $\bar{x} \pm t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}$

Confidence Interval for a Proportion: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Sample Size (Mean): $n = (\frac{z_{\alpha/2}\sigma}{E})^2$

Sample Size (Proportion): $n = (\frac{z_{\alpha/2}}{E})^2 p(1-p)$

Sample Size (Proportion, not specified): $n = (\frac{z_{\alpha/2}}{E})^2 0.5^2$

Standard Normal Probabilities

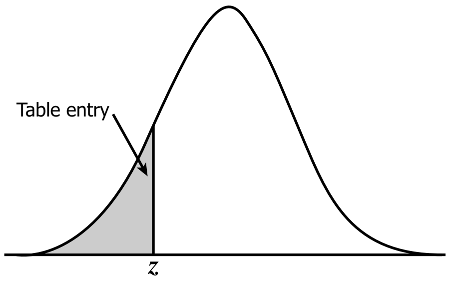

Table entry for z is the area under the standard normal curve to the left of z.

z	**.00**	**.01**	**.02**	**.03**	**.04**	**.05**	**.06**	**.07**	**.08**	**.09**
–3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
–3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
–3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
–3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
–3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
–2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
–2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
–2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
–2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
–2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
–2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
–2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
–2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
–2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
–2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
–1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
–1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
–1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
–1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
–1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
–1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
–1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
–1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
–1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
–1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
–0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
–0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
–0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
–0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
–0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
–0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
–0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
–0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
–0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
–0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

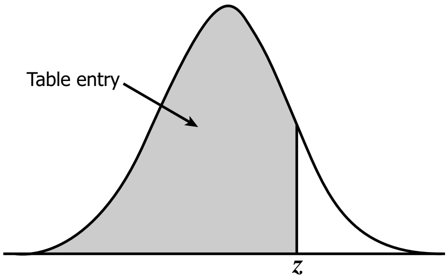

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										