

## Final Formula Sheet

### Probability Rule:

Probability of Union (Two):  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Probability of Union (Three):  $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

Conditional Probability:  $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Independence:  $P(A \cap B) = P(A) \times P(B)$  or  $P(B|A) = P(B)$

Total Probability Rule: Suppose  $E_1, E_2, \dots, E_k$  are k exhaustive and mutually exclusive events, then  $P(B) = P(B \cap E_1) + P(B \cap E_2) + \dots + P(B \cap E_k) = P(B|E_1)P(E_1) + \dots + P(B|E_k)P(E_k)$

### Discrete R.V.:

Mean (Expected Value):  $\mathbb{E}(X) = \mu = \sum_x x f(x)$

Variance:  $\mathbb{V}(X) = \sigma^2 = \sum_x (x - \mu)^2 f(x) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$

C.D.F:  $F(x) = P(X \leq x) = \sum_{y:y < x} f(x)$

### Continuous R.V.:

Mean (Expected Value):  $\mathbb{E}(X) = \mu = \int_{-\infty}^{\infty} x f(x) dx$

Variance:  $\mathbb{V}(X) = \sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$

C.D.F:  $F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt$

### Common Distributions:

Binomial Distribution (n,p):

$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$ ,  $x = 0, 1, 2, \dots, n$

$\mathbb{E}(X) = np$ ,  $V(X) = np(1-p)$

Hypergeometric Distribution (n,K,N):

$f(x) = \frac{\binom{K}{x} \binom{N-K}{n-x}}{\binom{N}{n}}$ ,  $x = \max\{0, n+K-N\}$  to  $\min\{K, n\}$

$\mathbb{E}(X) = np$ ,  $V(X) = np(1-p) \frac{N-n}{N-1}$ , where  $p = \frac{K}{N}$

Geometric Distribution (p):

$f(x) = p(1-p)^{x-1}$ ,  $x = 0, 1, 2, \dots$

$\mathbb{E}(X) = \frac{1}{p}$ ,  $V(X) = \frac{1-p}{p^2}$

Negative Binomial Distribution (r,p):

$f(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$ ,  $x = 0, 1, 2, \dots$

$\mathbb{E}(X) = \frac{r}{p}$ ,  $V(X) = \frac{r(1-p)}{p^2}$

Poisson Distribution ( $\lambda$ ):

$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$ ,  $x = 0, 1, 2, \dots$

$\mathbb{E}(X) = \lambda$ ,  $V(X) = \lambda$

Normal Distribution ( $\mu, \sigma^2$ ):

$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$ ,  $-\infty < x < \infty$

$\mathbb{E}(X) = \mu$ ,  $V(X) = \sigma^2$

Exponential Distribution ( $\lambda$ ):

$f(x) = \lambda e^{-\lambda x}$ ,  $x \geq 0$

$F(x) = P(X \leq x) = 1 - e^{-\lambda x}$

$\mathbb{E}(X) = \frac{1}{\lambda}$ ,  $V(X) = \frac{1}{\lambda^2}$

Normal Approximation to Binomial Distribution:

$$P(X \leq x) = P(X \leq x + 0.5) \approx P(Z \leq \frac{x + 0.5 - np}{\sqrt{np(1-p)}}$$

$$P(X \geq x) = P(X \geq x - 0.5) \approx P(Z \geq \frac{x - 0.5 - np}{\sqrt{np(1-p)}}$$

Normal Approximation to Poisson Distribution:

$$P(X \leq x) = P(X \leq x + 0.5) \approx P(Z \leq \frac{x + 0.5 - \lambda}{\sqrt{\lambda}}$$

$$P(X \geq x) = P(X \geq x - 0.5) \approx P(Z \geq \frac{x - 0.5 - \lambda}{\sqrt{\lambda}}$$

Marginal Distributions:  $f_X(x) = \int f_{XY}(x, y)dy$ ,  $f_Y(y) = \int f_{XY}(x, y)dx$

Covariance:  $Cov(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)$

Correlation:  $\rho_{XY} = \frac{Cov(X, Y)}{\sqrt{V(X)V(Y)}} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$

Standardizing:  $Z = \frac{X - \mu}{\sigma}$

Sample Mean:  $\bar{X} = \frac{\sum_{i=1}^n x_i}{n}$

Sample Variance:  $s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1} = \frac{\sum_{i=1}^n x_i^2 - n\bar{x}^2}{n-1}$

$Q_1$ : The  $(n+1)/4$  th number in the data set.

$Q_3$ : The  $3(n+1)/4$  th number in the data set.

Outliers:  $Q_1 - 1.5IQR, Q_3 + 1.5IQR$

Normal Probability Plot:  $\Phi(z_j) = \frac{j-0.5}{n}$ ,  $j = 1, 2, \dots, n$

Central Limit Theorem Formula:  $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

z-Confidence Interval for the Mean:  $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

t-Confidence Interval for the Mean:  $\bar{x} \pm t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$

Confidence Interval for a Proportion:  $\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Sample Size (Mean):  $n = (\frac{z_{\alpha/2}\sigma}{E})^2$

Sample Size (Proportion):  $n = (\frac{z_{\alpha/2}}{E})^2 p(1-p)$

Sample Size (Proportion, not specified):  $n = (\frac{z_{\alpha/2}}{E})^2 0.5^2$

Z Test for Mean:  $Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

Two Tail Beta:  $\beta = P(Z \leq z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}) - P(Z \leq -z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma})$  where  $\delta = \mu - \mu_0$

One Tail Beta ( $\mu > \mu_0$ ):  $\beta = P(Z \leq z_{\alpha} - \frac{\delta\sqrt{n}}{\sigma})$

One Tail Beta ( $\mu < \mu_0$ ):  $\beta = P(1 - (Z \leq -z_{\alpha} - \frac{\delta\sqrt{n}}{\sigma}))$

Sample Size (two tail):  $n = \sigma^2 \frac{(z_{\alpha/2} + z_{\beta})^2}{\delta^2}$

T Test for Mean:  $T = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$

Z Test for Proportions:  $Z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$

Two Tail Beta:  $\beta = P(Z \leq \frac{p_0 - p + z_{\alpha/2} \sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}) - P(Z \leq \frac{p_0 - p - z_{\alpha/2} \sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}})$

Confidence Interval for Difference in Means:

Variances Equal:  $\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2, n_1+n_2-2} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$  where  $s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$

Variances Unequal:  $\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2, v} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$  where  $v = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{(s_1^2/n_1)^2/(n_1-1) + (s_2^2/n_2)^2/(n_2-1)}$

Simple Regression Model:  $b_0 = \bar{y} - b_1\bar{x}$ ,  $b_1 = \frac{S_{xy}}{S_{xx}}$  where  $S_{xy} = \sum x_i y_i - n\bar{x}\bar{y}$  and  $S_{xx} = \sum x_i^2 - n(\bar{x})^2$

T Test for  $\beta_1$ :  $t_{n-2} = \frac{b_1}{\sqrt{\hat{\sigma}^2/S_{xx}}}$  where  $\hat{\sigma}^2 = \frac{SSE}{n-2}$

Residual Sum of Squares:  $SSE = \sum_{i=1}^n (y_i - \hat{y}_i)^2$

Regression Sum of Squares:  $SSR = \sum_{i=1}^n (\hat{y}_i - \bar{y})^2 = b_1 S_{xy}$

Total Sum of Squares:  $SST = \sum_{i=1}^n (y_i - \bar{y})^2 = \sum_i y_i^2 - n\bar{y}^2$

Confidence Interval for  $b_1$ :  $b_1 \pm t_{n-2, \alpha/2} \frac{\hat{\sigma}}{\sqrt{S_{xx}}}$

Prediction Interval for Mean:  $\hat{y} \pm t_{n-2, \alpha/2} \hat{\sigma} \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}}$

Prediction Interval for Individual:  $\hat{y} \pm t_{n-2, \alpha/2} \hat{\sigma} \sqrt{1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}}$

Sample Correlation Coefficient:  $r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$ , Coefficient of Determination:  $R^2 = \frac{SSR}{SST}$

Total Sum of Squares:  $SST = \sum_{i=1}^a \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{..})^2 = \sum_{i=1}^a \sum_{j=1}^{n_i} y_{ij}^2 - \frac{y_{..}^2}{N}$ ,  $df = N - 1$

Error Sum of Squares:  $SSE = \sum_{i=1}^a \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{i.})^2 = \sum_{i=1}^a (n_i - 1) s_i^2$ ,  $df = N - a$

Treatment Sum of Squares:  $SSTr = \sum_{i=1}^a n_i (\bar{y}_{i.} - \bar{y}_{..})^2 = \sum_{i=1}^a \frac{y_{i.}^2}{n_i} - \frac{y_{..}^2}{N}$ ,  $df = a - 1$

Fisher's LSD Test:  $LSD = t_{\alpha/2, N-a} \sqrt{MSE \left( \frac{1}{n_i} + \frac{1}{n_j} \right)}$

CI for LSD:  $\bar{y}_{i.} - \bar{y}_{j.} \pm LSD$

## Standard Normal Probabilities

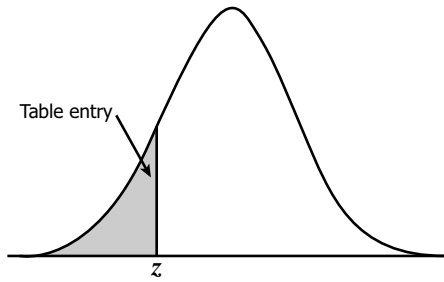


Table entry for  $z$  is the area under the standard normal curve to the left of  $z$ .

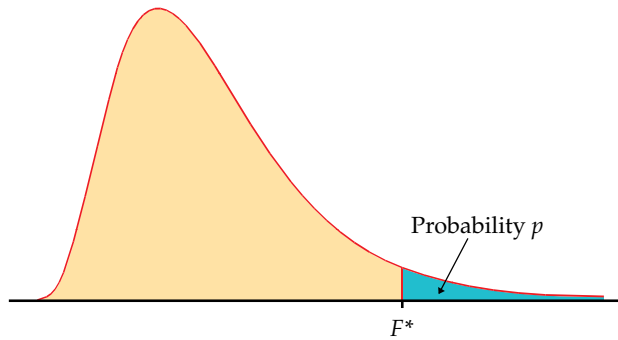
$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



# t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	<b>0.50</b>	<b>0.25</b>	<b>0.20</b>	<b>0.15</b>	<b>0.10</b>	<b>0.05</b>	<b>0.025</b>	<b>0.01</b>	<b>0.005</b>	<b>0.001</b>	<b>0.0005</b>
two-tails	<b>1.00</b>	<b>0.50</b>	<b>0.40</b>	<b>0.30</b>	<b>0.20</b>	<b>0.10</b>	<b>0.05</b>	<b>0.02</b>	<b>0.01</b>	<b>0.002</b>	<b>0.001</b>
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
<b>Z</b>	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	<b>Confidence Level</b>										

Table entry for  $p$  is the critical value  $F^*$  with probability  $p$  lying to its right.



**TABLE E**  
**F critical values**

		Degrees of freedom in the numerator									
$p$		1	2	3	4	5	6	7	8	9	
Degrees of freedom in the denominator	1	.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
		.050	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
		.025	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28
		.010	4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5
		.001	405284	500000	540379	562500	576405	585937	592873	598144	602284
	2	.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
		.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
		.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39
		.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
		.001	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37	999.39
	3	.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
		.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
		.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47
		.010	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
		.001	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86
	4	.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
		.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
		.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90
		.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
		.001	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47
	5	.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
		.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
		.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
		.010	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
		.001	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	27.24
	6	.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
		.050	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
		.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
		.010	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
		.001	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69
	7	.100	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
		.050	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
		.025	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
		.010	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
		.001	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33