Math 1M03 (Summer 2015) Sample Test #2

Name:_____
(Last Name) (First Name)

Student Number:_____

This test consists of 20 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. Questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Only the McMaster standard calculator Casio fx-991 is allowed.

1.) Suppose that $\int_1^3 f(x)dx = 5$ and $\int_1^3 g(x)dx = -2$.
Evaluate $\int_1^3 [2f(x) + 3g(x)]dx$

(a) 4 (b) 5 (c) 6 (d) 7 (e) 8

2.) Find the area bounded by the curves $y = x^2 - 4$ and $y = -x^2 + 4$

(a) $\frac{50}{3}$ (b)$\frac{64}{3}$ (c) $\frac{60}{3}$ (d) $\frac{56}{3}$ (e) $\frac{53}{3}$

3.) Using integration by parts, which of the following is equal to

$$\int x^5 e^x dx?$$

(a) $x^5e^x - 5\int x^4e^x dx$(b) $x^4e^x - 5\int x^5e^x dx$ (c) $\frac{1}{5}x^5e^x - 5\int x^4e^x dx$
(d) $\frac{1}{4}x^4e^x - 5\int x^4e^x dx$ (e)$\frac{1}{5}x^5e^x - 4\int x^4e^x dx$

4.) Evaluate the following integral

$$\int_0^1 \frac{3x^4 + 2x^3}{x^2} dx$$

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{2}$ (d) 2 (e) 3

5.) Evaluate the following integral

$$\int x \ln x dx$$

(a) $\frac{1}{2}x\ln|x| + \frac{1}{4}x^2 + C$ (b) $\frac{1}{4}x^2\ln|x| + \frac{1}{2}x + C$ (c) $\frac{1}{2}x^2\ln|x| + \frac{1}{6}x^3 + C$
(d) $\frac{1}{4}x\ln|x| + \frac{1}{2}x^3\ln|x| + C$ (e) $\frac{1}{2}x^2\ln|x| - \frac{1}{4}x^2 + C$

6.) Find the area of the region R that lies under the curve $y = e^{3x}$ over the interval $0 \leq x \leq \ln 2$

(a) $\frac{10}{3}$ (b)2 (c)3 (d) $\frac{8}{3}$ (e) $\frac{7}{3}$

7.) The population of a certain community t years after the year 2000 is given by

$$P(t) = \frac{e^{.8t}}{4 + e^{.8t}}$$

million people.What was the average population of the community during the period from 2000 to 2007?
(a) 1,235,261 (b) 568,912 (c) 438,217 (d) 715,222 (e) 347,265

8.) Which of the following represents the area between the curves $y = x^2$ and $y = x$ on the interval $0 \leq x \leq 2$?

(a) $\int_0^2 (x - x^2)dx$(b) $\int_0^1 (x - x^2)dx + \int_1^2 (x^2 - x)dx$ (c) $\int_0^2 (x^2 - x)dx$
(d) $\int_0^1 (x^2 - x)dx + \int_1^2 (x - x^2)dx$ (e) $\int_0^2 (\frac{1}{3}x^3 - \frac{1}{2}x^2)dx$

9). Evaluate the following improper Integral

$$\int_1^\infty xe^{-2x^2}dx$$

(a) $-\frac{1}{2e^2}$(b) e^{-2} (c) $\frac{3}{4e^2}$ (d) $\frac{1}{4e^2}$ (e) diverges

10). Evaluate the following improper Integral

$$\int_{e^2}^\infty \frac{2}{x(\ln x)^2}dx$$

(a) 1 (b) $-\frac{1}{e^4}$ (c) $\frac{2}{e^2}$ (d) $\frac{1}{4}$ (e) diverges

11.) Find the domain of the function $f(x, y) = \ln(3x^2 + 6y + 1)$.

(a) $y \geq -\frac{1+3x^2}{6}$ (b)$y > -\frac{1+3x^2}{6}$ (c) $y \leq \frac{1+3x^2}{6}$ (d) $y < \frac{1+3x^2}{6}$ (e) $y < -\frac{1+3x^2}{6}$

12.) Find $g_y(x, y)$ if

$$g(x, y) = \frac{yx^2}{\sqrt{1 + y}}$$

(a) $\frac{2xy\sqrt{1+y}+x^2y\sqrt{1+y}-\frac{1}{2}x^2y(1+y)^{3/2}}{1+y}$ (b) $\frac{(2+y)x^2}{2(1+y)^{3/2}}$ (c) $2\ln(xy) - \frac{1}{2}\ln y$
(d) $4x\sqrt{1+y} + x^2\sqrt{1+y}$ (e) $\frac{2y}{3(1+y)^{3/2}}$

13.) Evaluate $f_{xy}(8,4)$ if

$$f(x,y) = x\sqrt{y} - y\sqrt{x-y}$$

(a) -2 (b) $\frac{1}{2}$ (c) $-\frac{1}{16}$ (d) $\frac{7}{32}$ (e) $-\frac{1}{8}$

14.) Let

$$h(u,v) = ue^{u^2+v^3}$$

Compute $h_u(2,1)$

(a) e^5(b) $2e^5$ (c) $4e^5$ (d) $9e^5$(e) $8e^5$

15.) A function $f(x,y)$ has derivatives

$$f_x(x,y) = \ln(\frac{y^2}{x})+2-y^2, f_y(x,y) = \frac{2x}{y}-2xy, f_{xx}(x,y) = -\frac{1}{x}, f_{yy}(x,y) = -\frac{2x}{y}-2x, f_{xy}(x,y) = \frac{2}{y}-2y$$

Then we can say that the point $(e,1)$ is

(a) a critical point and a local minimum
(b) not a critical point but a local maximum
(c) a critical point and a local maximum
(d) not a critical point but a local minimum
(e) a critical point and a saddle

16.) For what values of the exponent p does the following integral converge (i.e. not diverge)?

$$\int_1^\infty \frac{3x}{x^p}dx$$

(a) $p > 1$ (b) $p \leq 1$ (c) $p > 2$ (d) $p \geq 2$ (e) $p < 1$

17.) Find the critical points of the function

$$f(x,y) = e^{y^3-x^3+6xy}$$

(a) $(0,0)$ only (b) $(-2,2)$ only (c) $(0,0)$ and $(-2,2)$ (d) $(0,0)$ and $(3,\ln 2)$ (e) $(3,\ln 2)$ only

18.) If x is the number of thousands of dollars spent on labour, and y is the thousands of dollars spent on parts, then the output of a factory is given by:

$$Q(x,y) = 42x^{1/4}y^{3/4}$$

where $Q(x,y)$ is the output in millions of units of product. Now, if \$200,000 is to be spent on parts and labour, how much should be spent on each to optimize

output?

(a) \$150,000 on parts and \$50,000 on labour
(b) \$75,000 on parts and \$125,000 on labour
(c) \$100,000 on parts and \$100,000 on labour
(d) \$50,000 on parts and \$150,000 on labour
(e) \$125,000 on parts and \$75,000 on labour

19.) Consider the function

$$g(x,y) = e^{3x^2-4y^2+24y}$$

Find and classify (if possible) the critical point of g.

(a) $(0,3)$, saddle point (b) $(0,3)$, relative minimum
(c) $(0,3)$, relative maximum (d) $(0,3)$, inconclusive (e) $(2,0)$, relative minimum

20.) Let $z = f(x,y) = \frac{\sqrt{x}}{y}$, where $x = 4t^2$and $y = t^2+3t+1$. Find the value of $\frac{dz}{dt}$ when $t = 1$.

(a) 2 (b) -2 (c) 1 (d) 0 (e) -1

21.) Correctly fill out the bubbles corresponding to your student number and the version number of your test in the correct places on the computer card.

Solutions:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
a	b	a	d	e	e	d	b	d	a	b	b	e	d	c	c	c	a	a	d