Math 1M03 (Version 1)
Sample Exam

Name:____
(Last Name) (First Name)

Student Number:______________________

Day Class
Duration: 3 Hours
Maximum Mark: 40

McMaster University Sample Final Examination

This examination paper consists of 15 pages (including this one). This exam consists of 38 multiple choice questions worth 1 mark each (no part marks). The questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Only the McMaster standard calculator Casio fx-991 is allowed.

Computer Card Instructions:

NOTE: **IT IS YOUR RESPONSIBILITY TO ENSURE THAT THE ANSWER SHEET IS PROPERLY COMPLETED: YOUR EXAMINATION RESULT DEPENDS UPON PROPER ATTENTION TO THESE INSTRUCTIONS**

The scanner, which reads the sheets, senses the shaded areas by their non-reflection of light. A heavy mark must be made, completely filling the circular bubble, with an HB pencil. Marks made with a pen or felt-tip marker will **NOT** be sensed. Erasures must be thorough or the scanner may still sense a mark. Do **NOT** use correction fluid on the sheets. Do **NOT** put any unnecessary marks or writing on the sheet.

1. Print your name, student number, course name, and the date in the space provided at the top of Side 1 (red side) of the form. Then the sheet **MUST** be signed in the space marked SIGNATURE.

2. Mark your student number in the space provided on the sheet on Side 1 and fill in the corresponding bubbles underneath.
3. Mark only **ONE** choice from the alternatives (A,B,C,D,E) provided for each question. If there is a True/False question, enter response of 1 (or A) as True, and 2 (or B) as False. The question number is to the left of the bubbles. Make sure that the number of the question on the scan sheet is the same as the question number on the test paper.
4. Pay particular attention to the Marking Directions on the form.
5. Begin answering questions using the first set of bubbles, marked "1".

1. Evaluate the following integral,

$$\int_1^e x \ln x \, dx$$

(a) $\frac{1}{4}(2e^2+1)$ **(b)** $\frac{1}{4}$ **(c)** $\frac{1}{4}(e^2+1)$ **(d)** e^2+1 **(e)** e^2+2

2. Evaluate the following integral,

$$\int_0^\infty \frac{2e^x}{(1+e^x)^2} \, dx$$

(a) 0 **(b)** 1 **(c)** 2 **(d)** e^{-1} **(e)** $-2e^{-1}$

3. At what point(s) does the following function attain its maximum, subject to the constraint $x^2+y^2=12$?

$$f(x,y) = x^{4/3}y^{2/3}$$

(a) $(1,-1),(-1,1)$ **(b)** $(2\sqrt{2},2),(2\sqrt{2},-2)$ only **(c)** $(\sqrt{6},-\sqrt{6}),(\sqrt{6},\sqrt{6})$ only
(d) $(2\sqrt{2},2),\ (2\sqrt{2},-2),\ (-2\sqrt{2},2),(-2\sqrt{2},-2)$
(e) $(\sqrt{6},-\sqrt{6}),(\sqrt{6},\sqrt{6}),(-\sqrt{6},-\sqrt{6}),(-\sqrt{6},\sqrt{6})$

4. Find the mean μ of the random variable X with the following probability density function

$$f(x) = \begin{cases} \frac{1}{4\sqrt{x}} & 4 \le x \le 16 \\ 0 & \text{otherwise} \end{cases}$$

(a) $\frac{28}{3}$ **(b)** 9 **(c)** $\frac{26}{3}$ **(d)** $\frac{25}{3}$ **(e)** $\frac{29}{3}$

5. Given the function

$$f(x,y) = x^2+4xy+y^2$$

and the points

(i) $(0,0)$ **(ii)** $\left(-\frac{1}{4},\frac{1}{\sqrt{8}}\right)$ **(iii)** $\left(-\frac{1}{4},\frac{1}{\sqrt{8}}\right)$ **(iv)** $\left(-\frac{1}{4},0\right)$ **(v)** $(1,1)$

which are critical points?

(a) (i) and (ii) only **(b)** (i), (ii), and (iii) **(c)** (ii) and (iii) only **(d)** (i) and (iv) **(e)** (iv) only

6. Assuming that $a \neq 0$, find the average value of the function

$$f(x) = axe^{-ax}$$

on the interval $0 \leq x \leq 2$.
(a) $-e^{-2a}$ **(b)** $\frac{(e^{-2a}-1)}{2a}$ **(c)** $-e^{-2a} - \frac{(e^{-2a}-1)}{2a}$ **(d)** $-2e^{-2a} - \frac{(e^{-2a}-1)}{a}$ **(e)** $\frac{(e^{-2a}-1)}{a}$

7. Suppose that the random variable X has the following probability density function,

$$f(x) = \begin{cases} x^2 e^{-2x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Find $P(1 < X < 2)$.
(a) $\frac{3}{4}e^{-2} - \frac{7}{4}e^{-4}$ **(b)** $\frac{5}{4}e^{-1} - \frac{7}{4}e^{-2}$ **(c)** $\frac{13}{4}e^{-2} - \frac{5}{4}e^{-4}$ **(d)** $\frac{5}{4}e^{-2} - \frac{13}{4}e^{-4}$
(e) $\frac{13}{4}e^{-2} - \frac{1}{4}e^{-4}$

8. Find the constant k such that the following function is a probability density function.

$$f(x) = \begin{cases} \frac{k}{x(\ln x)^2} & x \geq 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) $2\ln 2$ **(b)** $\ln 5$ **(c)** e **(d)** e^{-1} **(e)** $\ln 2$

9. Let X be a random variable with probability density function

$$f(x) = \begin{cases} \frac{5}{4x^2} & 1 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

Find the variance of X.
(a) $5 - \frac{25}{16}(\ln 5)^2$ **(b)** $25 - \frac{5}{16}(\ln 5)^2$ **(c)** $16 - \frac{1}{5}(\ln 5)^2$ **(d)** $12 - \frac{25}{16}(\ln 5)^2$
(e) $16 - \frac{25}{16}(\ln 5)^2$

10. Evaluate the following integral,

$$\int_2^9 x^2(x-1)^{-1/3}\,dx$$

(a) $\frac{5497}{40}$ **(b)** $\frac{5493}{40}$ **(c)** $\frac{2747}{20}$ **(d)** $\frac{1099}{8}$ **(e)** $\frac{687}{5}$

11. Given that $z = x^2 y$, where $x = g(t)$, $y = f(t)$ and that $f(1) = 2$, $f'(1) = 3$, $g(1) = 1$, $g'(1) = 4$. Compute $\frac{dz}{dt}$ at $t = 1$.
(a) 48 **(b)** 5 **(c)** 19 **(d)** 12 **(e)** -7

12. Solve the following equation for x,

$$\ln x + 2\ln x^{1/2} + 5 - \ln(x-1) - \ln(2x) = 0$$

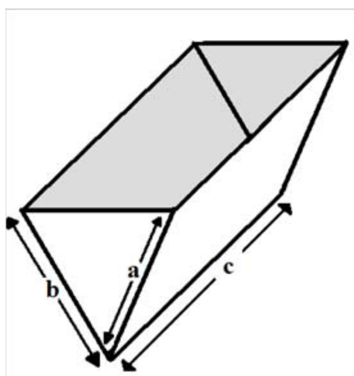
(a) $1 + e^5$ **(b)** $\sqrt{2 - e^5}$ **(c)** $\frac{1}{2}(2 + e^5)$ **(d)** $\frac{2}{2-e^5}$ **(e)** 0

13. Solve the following differential equation

$$y' = \frac{2\ln x}{x^3} - \frac{2y}{x}$$

(a) $\left(\frac{\ln x}{x}\right)^2 + \frac{C}{x}$ **(b)** $\frac{\ln x}{x^3} + Cx$ **(c)** $\frac{(\ln x)^2}{x^2} + C$ **(d)** $\frac{\ln x - 1 + Cx}{x^3}$ **(e)** $\left(\frac{\ln x}{x}\right)^2 + \frac{C}{x^2}$

14. An open-topped triangular prism (see below) is constructed from 300cm^2 of cardboard. Find the maximum volume of this open prism.
Hint: If the side panels are both c long and a, and b wide respectively, then the area of each end is given by: End Area = $ab/2$ (assuming that the angle between the sides with length a and b is 90°). And the volume is (length) x (end area).



(a) 500cm^3 **(b)** 100cm^3 **(c)** 360cm^3 **(d)** 60cm^3 **(e)** 96cm^3

15. Find the domain of the following function

$$f(x,y) = \sqrt{\ln(x+y)}$$

(a) $0 \leq x+y \leq 1$ **(b)** $x+y \geq 1$ **(c)** $x+y \neq 0$ **(d)** $x+y \geq 0$ **(e)** $x+y \geq e$

16. An investor deposits k dollars per year at a constant rate into an account earning interest at a fixed annual rate r compounded continuously. Suppose that $B(t)$ is the value of the account t years after the initial deposit. Then, which of the following differential equations best describes $\frac{dB}{dt}$?

(a) $\frac{dB}{dt} = B + ke^{rt}$ **(b)** $\frac{dB}{dt} = kB + e^{rt}$ **(c)** $\frac{dB}{dt} = rB + k$ **(d)** $\frac{dB}{dt} = r + k$
(e) $\frac{dB}{dt} = B + rk$

17. Sarah makes a single deposit of \$20,000 into a savings account which earns interest at a rate of 5% per year, compounded continuously. She plans to gradually withdraw money at a constant rate of \$3000 per year. To the nearest month, the account will be exhausted after how many years and months?
(a) 7 years and 8 months **(b)** 7 years and 11 months **(c)** 8 years and 5 months **(d)** 8 years and one month **(e)** 9 years and 2 months

18. Let $f(x) = x^2(\ln x^{1/2} + \ln x^{1/3} + \ln x^{1/4})$. Find the equation of the tangent line at $x = 1$.
(a) $y = \frac{12}{13}x - \frac{12}{13}$ **(b)** $y = \frac{12}{13}x - \frac{13}{12}$ **(c)** $y = \frac{13}{12}x - \frac{12}{13}$ **(d)** $y = x - \frac{12}{13}$
(e) $y = \frac{13}{12}x - \frac{13}{12}$

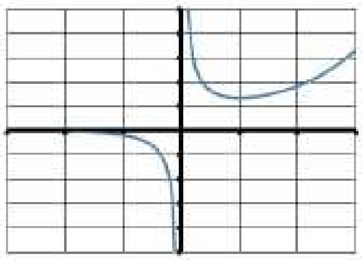
19. It is known that the following function has exactly one saddle point.

$$f(x, y) = x^3 + y^2 - 6xy + 9x + 5y + 2$$

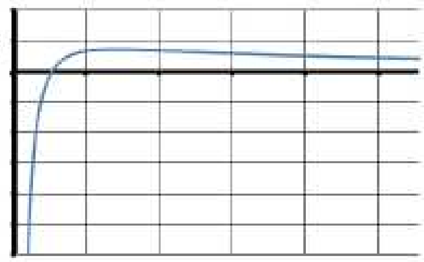
Find the saddle point.
(a) $(2, \frac{7}{2})$ **(b)** $(2, \frac{5}{2})$ **(c)** $(2, 4)$ **(d)** $(4, \frac{19}{2})$ **(e)** $(4, \frac{15}{2})$

20. Which of the following is the graph of $f(x) = \frac{\ln x}{x}$?

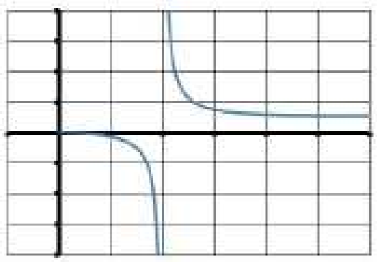
A



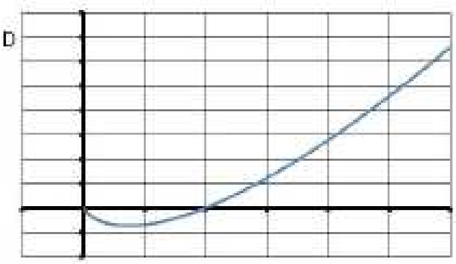
B



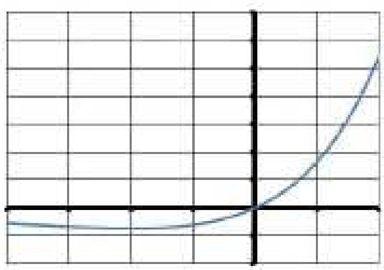
C



D



E



(a) **(b)** **(c)** **(d)** **(e)**

21. Given the function $f(x) = x^2$, consider the area under the curve $y = f(x)$ above the x-axis for $1 \leq x \leq a$. What is the value a such that this area will be equal to 1?
(a) 3 **(b)** $\sqrt{3}$ **(c)** $\sqrt[3]{4}$ **(d)** 2 **(e)** $\sqrt[3]{2}$

22. Assuming that the function $f(x)$ is *odd*, i.e., $f(x) = -f(-x)$, suppose that $\int_0^4 f(x)dx = 6$ and $\int_4^2 2f(x)dx = 3$. Then find the value of $\int_{-2}^0 f(x)dx$.
(a) $\frac{11}{2}$ **(b)** 6 **(c)** $-\frac{13}{2}$ **(d)** $-\frac{15}{2}$ **(e)** 7

23. In a tourist souvenir shop, the owner sells two types of T-shirt, A and B. It is estimated that if T-shirts A are sold at x dollars each and T-shirts B are sold at y dollars each, then $40 - 50x + 40y$ of T-shirts A and $20 + 60x - 70y$ of T-shirts B will be sold daily. Suppose that the cost of both T-shirts is \$2 dollars a piece. How should the owner price the T-shirts in order to maximize the daily profit?

(a) T-shirt A: \$2.8, T-shirt B: \$2.6 **(b)** T-shirt A: \$2.5, T-shirt B: \$2.5
(c) T-shirt A: \$2.6, T-shirt B: \$2.7 **(d)** T-shirt A: \$2.7, T-shirt B: \$2.6
(e) T-shirt A: \$2.7, T-shirt B: \$2.5

24. When x dollars are spent on labour and y dollars are spent on equipment, the output in a certain factory is given by

$$Q(x, y) = x^{1/3}y^{2/3}$$

Suppose that the budget is \$12,000 which is to be entirely spent on labour and equipment. What is the maximum possible output under this budget? (Round your answer to the nearest integer.)
(a) 6350 **(b)** 6320 **(c)** 6330 **(d)** 6340 **(e)** 6360

25. Glaucoma is a disease of the eye that is manifested by high intraocular pressure. The distribution of intraocular pressure in the general population is approximately normal with mean 16 mm Hg and standard deviation 3 mm Hg. What percentage of people have an intraocular pressure lower than 14 mm Hg?
(a) 36.27 **(b)** 25.14 **(c)** 28.65 **(d)** 31.14 **(e)** 33.97

26. Glaucoma is a disease of the eye that is manifested by high intraocular pressure. The distribution of intraocular pressure in the general population is approximately normal with mean 16 mm Hg and standard deviation 3 mm Hg. Find the intraocular pressure for which 80% of the population have an intraocular pressure that is greater than this value
(a) 10.53 **(b)** 12.87 **(c)** 13.48 **(d)** 14.55 **(e)** 8.62

27. Solve the following equation for x.

$$2\log_{3x}3 + \log_{3x}2 = 1$$

(a) 6 **(b)** 3 **(c)** 2 **(d)** 4 **(e)** 8

28. Let

$$f(x, y) = xe^{xy}$$

Find $f_{xy}(1, 0)$
(a) 0 **(b)** e **(c)** 1 **(d)** 3 **(e)** 2

29. Find the area enclosed between the graphs of $y = e^{2x}$, $y = e^{-3x}$, and $x = \ln 2$.
(a) $\frac{29}{24}$ **(b)** $\frac{9}{8}$ **(c)** 12 **(d)** $\frac{1}{2}e^2 + \frac{1}{3e^2} - \frac{5}{6}$ **(e)** $\frac{1}{3}e^2 + \frac{1}{2e^2} - \frac{6}{5}$

30. Which of the given points is in the domain of both functions

$$Q(x,y) = \frac{1}{\sqrt{25-4x^2-3y^2}}, \qquad P(x,y) = \ln(x^2+y^2-4)$$

(a) $(0,0)$ **(b)** $(1,\frac{5}{2})$ **(c)** $(0,-2)$ **(d)** $(-2,2)$ **(e)** $(1,1)$

31. It is found that a chemical reaction proceeds at a rate given by the equation

$$C'(t) = 90 + 9t^2 + (6-t)^5$$

Here, the rate is measured in grams per second, and time is measured in seconds. If at the start of the reaction ($t=0$) there are 100 grams of the chemical, approximately how much of the chemical do we have when $t=2$?
(a) 215g **(b)** 479g **(c)** 7,397g **(d)** 8,912g **(e)** 158g

32. Find the solution of the following differential equation,

$$y' + 2y^2x = 4y^2$$

given that $y=1$ when $x=0$.
(a) $y = \frac{1}{x^2-4x+C}$ **(b)** $y = \frac{1}{x^2-4x}+1$ **(c)** $y = \frac{1}{x^2-4x}+C$ **(d)** $y = \frac{1}{x^2-4x+1}$
(e) $y = \ln(x^2-4x+e)$

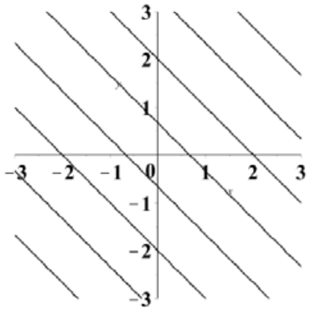
33. Evaluate the following improper integral,

$$\int_0^\infty xe^{-3x^2}\,dx$$

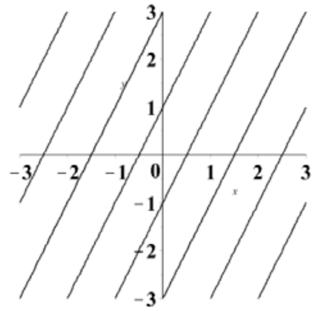
(a) diverges **(b)** 1 **(c)** $\frac{1}{2}$ **(d)** $\frac{1}{3}$ **(e)** $\frac{1}{6}$

34. Which of the following shows graphs of level sets (level curves, contour lines) of the function $f(x, y) = y + x - 2$?

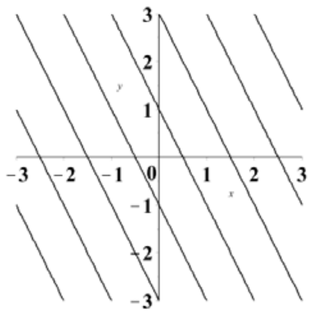
(a)



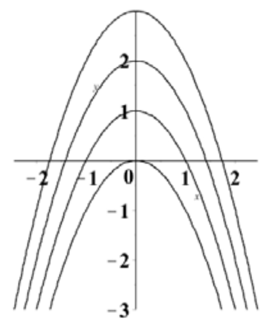
(b)



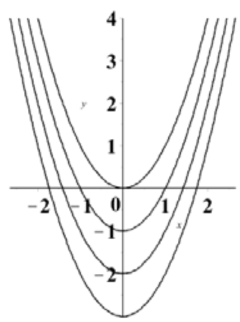
(c)



(d)



(e)



35. A medical researcher determines that t hours from the time a toxin is introduced to a bacterial colony, the population will be

$$P(t) = 10{,}000(7 + 15e^{-.05t} + te^{-.05t})$$

What is the maximum bacterial population?
(a) 184,391 **(b)** 225,760 **(c)** 287,215 **(d)** 358,219 **(e)** 556,121

36. A medical researcher determines that t hours from the time a toxin is introduced to a bacterial colony, the population will be

$$P(t) = 10{,}000(7 + 15e^{-.05t} + te^{-.05t})$$

What eventually happens to the bacterial population as $t \to \infty$?
(a) $P(t)$ approaches ∞ **(b)** $P(t)$ approaches 220,000 **(c)** $P(t)$ approaches 230,000
(d) $P(t)$ approaches 10,000 **(e)** $P(t)$ approaches 70,000

37. Simplify the following expression,

$$e^{-2\ln x} + \ln(\ln e^{e^x})$$

(a) $\frac{1}{x^2} + e^x$ **(b)** $\frac{1}{x^2} + x$ **(c)** $-x$ **(d)** $\frac{x}{e^2} + x$ **(e)** $\frac{x}{e^2} + e^x$

38. If Z is a standard normal random variable, find the value of b such that $P(-b < Z < b) = .7242$.
(a) 1.09 **(b)** 1.35 **(c)** 1.05 **(d)** 1.27 **(e)** 0.84

39. A hot drink is taken outside on a cold winter day when the air temperature is -6°C. According to a principle of physics called Newton's Law of Cooling, the temperature T (in degrees Celsius) of the drink t minutes after being taken outside is given by

$$T(t) = -6 + Ae^{-kt}$$

where A and k are constants. Suppose that the temperature of the drink is 83°C when it is taken outside and that 20 minutes later the drink is 26°C. When (i.e., after how many minutes) will the temperature reach 0°C?

(a) 63.24 **(b)** 68.15 **(c)** 55.81 **(d)** 52.73 **(e)** 58.24

40. Find the general solution to the following differential equation,

$$y' = \frac{x^6 y}{(x^7 + 8)^2}$$

(a) $y = Ce^{-1/[6(x^7+8)]}$ **(b)** $y = C(x^7 + 8)^{1/6}$ **(c)** $y = Ce^{-1/[7(x^7+8)]}$
(d) $y = C(x^7 + 8)^{1/7}$ **(e)** $y = (x^7 + 8)^{1/7} + C$

(THE END)

Answers:

1. c **2.** b **3.** d **4.** a **5.** b **6.** c **7.** d **8.** e **9.** a **10.** b
11. c **12.** d **13.** e **14.** a **15.** b **16.** c **17.** d **18.** e **19.** a **20.** b
21. c **22.** d **23.** e **24.** a **25.** b **26.** c **27.** a **28.** e **29.** a **30.** b
31. c **32.** d **33.** e **34.** a **35.** b **36.** e **37.** b **38.** a **39.** d **40.** c