

# Tutorial 4 Problems

## Probability

Oct. 7

### Suggested Problem 1

You have 12 red balls and 12 blue balls to distribute among 3 urns. You can distribute them among the urns any way that you want (the urns do not have to have equal numbers of balls in them). How would you distribute them to maximize the possibility that a random ball drawn from a randomly chosen urn is blue, and what is that probability?

### Suggested Problem 2

A man is at a park where he's just had to deal with his complaining daughter. He sits on a bench beside a woman and strikes up a conversation:

- a. Man: Have you any kids? Woman: Yes, 2.  
What's the probability that she has 2 girls?
- b. Man: Any girls? Woman: Yes.  
What's the probability that she has 2 girls?  $1/3$
- c. Just then a little girl runs up to the woman saying "Mommy! Mommy!"  
What's the probability that that girl's sibling is also a girl?

### Page 68 - Example 3i

An urn contains two type  $A$  coins and one type  $B$  coin. When a type  $A$  coin is flipped, it comes up heads with probability  $1/4$ , whereas when a type  $B$  coin is flipped, it comes up heads with probability  $3/4$ . A coin is randomly chosen from the urn and flipped. Given that the flip landed on heads, what is the probability that it was a type  $A$  coin?

### Page 70 - Example 3k

A plane is missing, and it is presumed that it was equally likely to have gone down in any of 3 possible regions. Let  $1-\beta_i$ ,  $i = 1, 2, 3$ , denote the probability that the plane will be found upon a search of the  $i^{th}$  region when the plane is, in fact, in that region. (The constants  $\beta_i$  are called overlook probabilities, because they represent the probability of overlooking the plane; they are generally attributable to the geographical and environmental conditions of the regions.) What is the conditional probability that the plane is in the  $i$ th region given that a search of region 1 is unsuccessful?

## Page 73 - Example 3n

A bin contains 3 different types of disposable flashlights. The probability that a type 1 flashlight will give over 100 hours of use is 0.7, with the corresponding probabilities for type 2 and type 3 flashlights being 0.4 and 0.3, respectively. Suppose that 20% of the flashlights in the bin are type 1, 30% are type 2, and 50% are type 3.

- a. What is the probability that a randomly chosen flashlight will give more than 100 hours of use?
- b. Given that a flashlight lasted over 100 hours, what is the conditional probability that it was a type  $j$  flashlight,  $j = 1, 2, 3$ ?

## Page 80 - Example 4i

Suppose there are  $n$  types of coupons and that each new coupon collected is independent of previous selections, a type  $i$  coupon with probability  $p_i$ ,  $\sum_{i=1}^n p_i = 1$ . Suppose  $k$  coupons are to be collected. If  $A_i$  is the event that there is at least one type  $i$  coupon among those collected, then, for  $i \neq j$ , find:

- a.  $P(A_i)$
- b.  $P(A_i \cup A_j)$
- c.  $P(A_i | A_j)$