## Textbook questions from chapter 5

4h, 5c, 6b, 7e
Refer to the textbook.

## Stock example from Dr. Lozinski's class: 5-13

A stock is $20 \$$. Every day, it increases by a tenth with probability $\frac{11}{20}$, and decreases by an eleventh with probability $\frac{9}{20}$.

What is the probability that it is above $30 \$$ after 100 days?

## solution:

Suppose $S$ is the current price. If it increases, then $S \rightarrow S+\frac{1}{10} S=\frac{11}{10} S$.
If the stock decreases then $S \rightarrow S-\frac{1}{11} S=\frac{10}{11} S$.
Let's let $u=\frac{11}{10}$. For example, if the stock prices increases, increases, then decreases over a 3 day period, we would have

$$
S u u u^{-1}=S \frac{u^{2}}{u}=S u
$$

meaning the stock price would be 1.1 times where it was originally (one increase would cancel out with one decrease).

Let's let $X=$ the number of increases over the 100 day period, thus $100-X$ is the number of decreases. Notice, this follows a binomial distribution where the event of an increase is our success, hence $p=\frac{11}{20}$, with $\mathrm{n}=100$.

We can model the final stock price with the following formula:

$$
20 u^{x}\left(u^{-1}\right)^{(100-x)}
$$

please note that this is NOT a probability distribution, we can simplify it to:

$$
20 u^{2 x-100}
$$

Now we can answer the final question by solving for

$$
P\left(20 u^{2 X-100}>30\right)
$$

isolating for x , (and remember $\mathrm{u}=1.1$, it's not a variable or anything):

$$
P\left(X>\frac{\frac{\log \frac{3}{2}}{\log (1.1)}+100}{2}\right)
$$

which evaluates approximately to

$$
P(X>52.1271)
$$

However, since we are approximating a discrete random variable with the continuous, we need to correct for that... it is better to extra careful, so we'll round that to

$$
P(X>52.5)
$$

Now, we will use the central limit theorem to turn X into Z so we can use the CDF of the standard normal distribution $\mathrm{N}(0,1)$.

$$
\begin{aligned}
& P\left(\frac{X-\mu}{\sigma}>\frac{52.5-\mu}{\sigma}\right) \\
& \quad=P\left(Z>\frac{52.5-\mu}{\sigma}\right)
\end{aligned}
$$

We know that $\mu=n p$ and $\sigma=\sqrt{n p(1-p)}$ from the binomial distribution.
So we get

$$
\mu=100 * 0.55=55, \sigma=\sqrt{100 * .55 * .45}=\frac{3 \sqrt{11}}{2} \doteq 5
$$

Subbing these in, we get $P(Z>-.5)$, however, we want to get this in the form of the CDF of the standardized normal, so we use the complement:

$$
1-P(Z \leq-.5)=1-\Phi(-.5)
$$

You should always draw yourself a picture (like we did in tutorial) to give yourself an estimate. We know the Z distribution is symmetric about 0 , so half of the probability will be found on each side about the $y$-axis. Here, we want $Z$ to be greater than -0.5 , this means we want all of the right half, and a piece of the left half, so we expect an answer greater than $50 \%$.

The $\Phi$ function is just notation that represents the CDF of the standard normal, that is, giving us the probability cumulatively from $-\infty$ to our value, in this case: -0.5 .

These values are typically read off of a chart (there is no closed-form solution) - as is most common during tests. The way you would do this is since you are given the QUANTILE (aka the value that tells you how much percent you want from the CDF), you would look that up along the sides of the chart, then pick out what probability is given to you. Different charts could give different answers, so make sure you look to see how they define their chart.

Certain softwares, such as $R$, can call upon this value more accurately than most charts:
\#Type the following code directly into the $R$ console
1-pnorm ( -.5 )
You should get a value close to $\mathbf{0 . 6 9 1 4 6 2 5}$ whichever method you use - meaning, the probability that after 100 days, there is a $69 \%$ chance that the stock, starting at $20 \$$ will climb to at least $30 \$$.

Notice, this is ONLY true following the somewhat restrictive assumptions that we've had to make along the way, however, this will be more of a concern in more advanced statistics courses.

