Assignment 1 / Due Wednesday June 27

PROBLEM 1   Show that the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2 + y^8}$$

PROBLEM 2   Show that the following limit is zero.

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x^2 + y^2}$$

PROBLEM 3   For the following function, find the partial derivatives, for all points $(x, y) \in \mathbb{R}^2$. Show that, the partial derivatives are not continuous at $(0,0)$. Prove by definition that, f is not differentiable at $(0,0)$.

$$f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$