Assignment 3 / Due Monday July 30

PROBLEM 1 Write the trigonometric Fourier series for $f(x) = x^2 + x$ on $(-\pi, \pi)$.

PROBLEM 2 Show that the following function has a local maximum at $(1, 1, 1)$.

$$f(x, y, z) = 4xyz - x^4 - y^4 - z^4$$

PROBLEM 3 Let $D \subset \mathbb{R}^3$ be a bounded open set with smooth boundary surface ∂D. Define the admissible class $\mathcal{A}_0$ of C^1 functions $u : \bar{D} \to \mathbb{R}$, which vanish on the boundary. Let $\vec{E} : \mathbb{R}^3 \to \mathbb{R}^3$ be a C^1 vector field, and consider the functional

$$I(u) = \iiint_D \frac{1}{2} |\nabla u - \vec{E}(x, y, z)|^2 dxdydz$$

(a) Expand out $I(u + tv)$ for $u, v \in \mathcal{A}_0$, in powers of t.
(b) Use part (a) to find the Euler-Lagrange equation.