Practice Problems

Problem 1 Find the critical points for the following functions and classify them.

$$\begin{aligned} i)\ f_1(x,y,z) &= xy + x^2z - x^2 - y - z^2 \\ ii)\ f_2(x,y,z) &= xyze^{-x^2-y^2-z^2} \\ iii)\ f_3(x,y) &= \left(1+\frac{1}{x}\right)\left(1+\frac{1}{y}\right)\left(\frac{1}{x}+\frac{1}{y}\right) \\ iv)\ f_4(x,y) &= xye^{-x^2-y^4} \\ v)\ f_5(x,y) &= \frac{1}{1-x+y+x^2+y^2} \end{aligned}$$

Problem 2 Practice problem 6, page 82 of the Courseware (chapter 7).

Problem 3 Consider the function $f : \mathbb{R}^3 \to \mathbb{R}$ defined by

$$f(x,y,z) = 4\sin x + (\pi - 2x)zy + e^y + 4z^2.$$

Show $(\frac{\pi}{2}, 0, 0)$ is a critical point, and determine whether it is a maximum, minimum or a saddle point.

Problem 5 Show $\mathcal{B} = \{P_n(x)\}$ is an orthogonal family with respect to $\langle f, g\rangle = \int_{-1}^{1} f(x)g(x)dx$, where

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$$

Find the Fourier series of the following functions with respect to $\mathcal{B}$:

$$\begin{aligned} (a)\ f(x) &= \begin{cases} 0 \text{ for } -1 < x < 0, \\ 1 \text{ for } 0 < x < 1, \end{cases} \\ (b)\ f(x) &= |x|. \end{aligned}$$