MATH 3MB3 FALL 2016 SAMPLE MIDTERM 1

Question 1. Consider the state variable $N(t)$ given by

$$N(t+1) = 1 + N(t) - \frac{1}{4}(N(t))^2.$$

a) Classify the model.

b) Find the fixed points.

c) Determine the stability of each fixed point.

Question 2. Consider $N(t)$ given by $N(t+1) = 1 - 2|N(t) - 0.5|$, where $|N(t) - 0.5|$ denotes the absolute value of $N(t) - 0.5$.

a) Classify the model.

b) Find the fixed points.

c) Determine the stability of each fixed point.

Hint: consider $N(t) > 0.5$ and $N(t) < 0.5$ separately.

Question 3. Consider $\vec{x}(t)$ determined by $\vec{x}(t+1) = A\vec{x}(t)$ where

$$\vec{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \quad A = \begin{bmatrix} 1/2 & -1 \\ a & -1/2 \end{bmatrix}.$$

Here a is a parameter. Assume now that $a \neq -3/4$.

a) Classify the model.

b) Find the fixed points.

c) Determine the stability of each fixed point.

d) Comment on the case $a = -3/4$.

Question 4. Consider $x(t)$ given by
$$\frac{dx}{dt} = 1 - x^2.$$

a) Classify the model.

b) Find its fixed points.

c) Determine the stability of each fixed point.

d) Find the solution $x(t)$ by solving the equation for an initial condition $x(0)$.

e) Make a phase diagram and confirm your conclusions about stability.