1. Question 1

16 marks for question 1
1a) $P(1) = 1.2(\frac{1}{2} - \frac{1}{20}) - \frac{1}{50} = \frac{13}{25} = 0.52$
1 mark
1b) $P(2) = 1.2(P(1) - \frac{1}{20}) - \frac{1}{50} = \frac{68}{120} = 0.544$
1 mark
1c) $P(t) = 1.2(P(t-1) - \frac{1}{20}) - \frac{1}{50}$
2 marks, -1 if signs are off but rest is ok
1d) linear univariate discrete deterministic (LUDD)
1 mark
1e) $P(1) = 1.2(P(0) - \frac{1}{20}) - \frac{1}{50}$
$P(2) = 1.2(1.2(P(0) - \frac{1}{20}) - \frac{1}{50} - \frac{1}{20}) - \frac{1}{50}$
$=1.2^2(P(0)) - \frac{1}{20}(1.2 + 1.2^2) - \frac{1}{50}(1 + 1.2)$
$=1.2^t(P(0)) - \frac{1}{20}\frac{1.2-1.2^{t+1}}{1-1.2} - \frac{1}{50}\frac{1-1.2^t}{1-1.2}$
1 mark for writing out first 1-2 steps
1 mark per correct term at the end (3 total)
-1 mark for wrong signs at the end
1f) Computer output that looks like:
0.5000 0.5200 0.5440 0.5728 0.6074 0.6488 0.6986 0.7583 0.8300 0.9160
1.0192 1.1430 1.2916 1.4699 1.6839 1.9407 2.2488 2.6186 3.0623 P(18)=3.0623
or if formated for long:
0.500000000000000 0.520000000000000 0.544000000000000 0.572800000000000
0.607360000000000 0.648832000000000 0.698598400000000 0.758318079999999
0.829981695999999 0.915978035199999 1.019173642239999 1.143008370687998
1.291610044825598 1.469932053790717 1.683918464548861 1.940702157458633
2.248842588950359 2.618611106740431 3.062333328088517 P(18)=3.062333328088517
Note for this question if their answer is 7.8795 or 7.879516567451061
(-1 mark) since they found year 17 not 18.
1 mark for stating what P(18) is
1 mark for showing the iteration (could be graphical as well)
1g) $P^* = 1.2(P^* - \frac{1}{20}) - \frac{1}{50}$
$0 = .2P^* - \frac{3}{50} - \frac{1}{50}$
$\frac{2}{25} = .2P^*$
$\frac{2}{5} = P^* = 0.4$
2 Marks for the ending

1 Mark for 1st line
1h)$R = 1.2 > 1$ therefore the fixed point is unstable (Note they could also use the derivative here to end up with this as well
2 Marks for conclusion

2. Question 2

6 marks for this question
2a) nonlinear univariate discrete deterministic (NUDD).
1 mark

2b) $S^* = S^* + (-\beta S^* + \gamma)(N - S^*)$
$S^* = N, \frac{\gamma}{\beta}$
2 marks
2c) $f' = 1 - \beta N + 2\beta S^* - \gamma$
$f'(N) = 1 + \beta N - \gamma$
N is stable if $-1 < 1 + \beta N - \gamma < 1$ or $-2 < \beta N - \gamma < 0$ (Note it's fine if they left it in absolute value form)
$f'(\frac{\gamma}{\beta}) = 1 - \beta N + \gamma$
$\frac{\gamma}{\beta}$ is stable if $-1 < 1 - \beta N + \gamma < 1$ or $-2 < -\beta N + \gamma < 0$
1 Mark for the derivative
1 mark for correct stability of each equilibrium (2 total)

3. Question 3

12 marks total
3a) It is a nonlinear univariate discrete deterministic model (NUDD)
1 mark
3b) Fixed points are $N^* = 0, L, K$
2 marks
3c) $f'(N) = 1 - R + \frac{2RN}{L} + \frac{2RN}{K} - \frac{3RN^2}{LK}$
$f'(0) = 1 - R$ which is stable if $-1 < 1 - R < 1$ or $-2 < -R < 0$ or $0 < R < 2$
$f'(L) = 1 - R + 2R + \frac{2RL}{K} - \frac{3RL}{K} = 1 + R - \frac{RL}{K}$
which is stable if $-1 < 1 + R - \frac{RL}{K} < 1$ or $-2 < R - \frac{R}{LK} < 0$
$f'(K) = 1 - R + \frac{2RK}{L} + 2R - \frac{3RK}{L} = 1 + R - \frac{3RK}{L}$
which is stable if $-1 < 1 + R - \frac{RK}{L} < 1$ or $-2 < R - \frac{RK}{L} < 0$
If they went further great, but not really needed/expected.
1 mark for derivative
1 mark for sticking in correct fixed points
3 marks for correct conclusion

3d) Fixed points are simply 0, 4, 8
They are simple just using 0,L,K from before but just plugging in the values via computer
1 mark
3e) Looking for a plot, or a chart, or a set of numbers.
Key part to actual look at (depending what their output is)
$N(0) = 1$ approaches $N^* = 0$ from above
$N(0) = 5$ approaches $N^* = 8$ from below
$N(0) = 11$ approaches $N^* = 8$ from above
1 mark for each