1. Eigenvector tips for 2D

If $M = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$
then determinant is $D = a_{11}a_{22} - a_{12}a_{21}$, trace is $T = a_{11} + a_{22}$, and eigenvalues obey $\lambda_1 + \lambda_2 = T$ and $\lambda_1\lambda_2 = D$. This leads to the characteristic polynomial $\lambda_i^2 - T\lambda_i + D$. And so the eigenvalues obey $\lambda_i = \frac{T \pm \sqrt{T^2 - 4D}}{2}$. Finally, if v_i and λ_i are an eigenvector/eigenvalue pair for M, then $Mv_i = \lambda_i v_i$ (i.e. a matrix and a single scalar value do the same thing to an eigenvector!).

2. geometric series explained

Why does $1 + R + R^2 + R^3 = \frac{1-R^4}{1-R}$?
Look at two infinite geometric series:
$1 + R + R^2 + R^3 + R^4 \dots$
$R^4 + R^5 + R^6 \dots$
The sum of the first series is $\frac{1}{1-R}$. The sum of the second series is $\frac{R^4}{1-R}$.
If we subtract the second from the first we end up with
$1 + R + R^2 + R^3$
Which is what we want and based off of the two sums above is evaluated as $\frac{1-R^4}{1-R}$.

3. example

$M = \begin{pmatrix} 2 & 7 \\ -1 & -6 \end{pmatrix}$
Find the eigenvalues and eigenvectors and hence $x(t)$, given that $x(0) = (-7\sqrt{\frac{1}{50}} + \sqrt{\frac{1}{2}}, \sqrt{\frac{1}{50}} - \sqrt{\frac{1}{2}})$
$T = -4, D = -5$ then
$\lambda_1 = \frac{-4 + \sqrt{(-4)^2 - 4*(-5)}}{2}$
$\lambda_1 = \frac{-4 + \sqrt{36}}{2}$
$\lambda_1 = \frac{-4+6}{2}$
$\lambda_1 = 1, \lambda_2 = -5$
Note that we can either solve $Av = \lambda v, (A - \lambda)v = 0$

$A - \lambda_1 = \begin{pmatrix} 1 & 7 \\ -1 & -7 \end{pmatrix}$
$v_1 = -7v_2$
$(-7v_2, v_2)$
If we want this to be length 1 then
$\sqrt{49v_2^2 + v_2^2} = 1$
$v_2 = \sqrt{\frac{1}{50}}$
$v_1 = -7\sqrt{\frac{1}{50}}$

$A - \lambda_2 = \begin{pmatrix} 7 & 7 \\ -1 & -1 \end{pmatrix}$
$v_1 = -v_2$
$v_1 = \sqrt{\frac{1}{2}}$
$v_2 = -\sqrt{\frac{1}{2}}$
$x(t) = c_1 d_1^t \vec{v_1} + c_2 d_2^t \vec{v_2}$
$x(0) = c_1 \vec{v_1} + c_2 \vec{v_2}$
$-7c_1\sqrt{\frac{1}{50}} + c_2\sqrt{\frac{1}{2}} = -7\sqrt{\frac{1}{50}} + \sqrt{\frac{1}{2}}$
$c_1\sqrt{\frac{1}{50}} - c_2\sqrt{\frac{1}{2}} = \sqrt{\frac{1}{50}} - \sqrt{\frac{1}{2}}$
Therefore $c_1 = 1$, $c_2 = 1$