

1. GRAPHICAL APPROACH

Use cobwebbing to understand the dynamics of (non)linear models. For linear models, choose an initial value  $N(0)$  and start the web at the point  $(N(0), N(0))$  on the diagonal. You then move vertically until you hit the graph of  $f$  (this is equivalent to calculating  $N(1)$  using the recursion equation). By doing so, you have traced out a straight line from the initial point to  $(N(1), N(0))$ . From there, the web drops back onto the diagonal in a horizontal sense, to the point  $(N(1), N(1))$ . The procedure is then repeated. The advantage to cobwebbing is that it reveals fixed points and shows convergence or divergence without much effort.

It is often a good idea to graph the quantity  $\Delta N(t) = N(t+1) - N(t)$  as a function of  $N(t)$ . Models that have  $\Delta N$  changing from negative to positive for increasing  $N$  are said to exhibit the Allee effect. For population models, for example, this corresponds to the idea that a sufficiently large population will boost survival rates (rather than doing the opposite due to overcrowding). Other phenomena, such as bistability and multiple stable states, occur for nonlinear models. These images are from Dr. Bolker's notes.

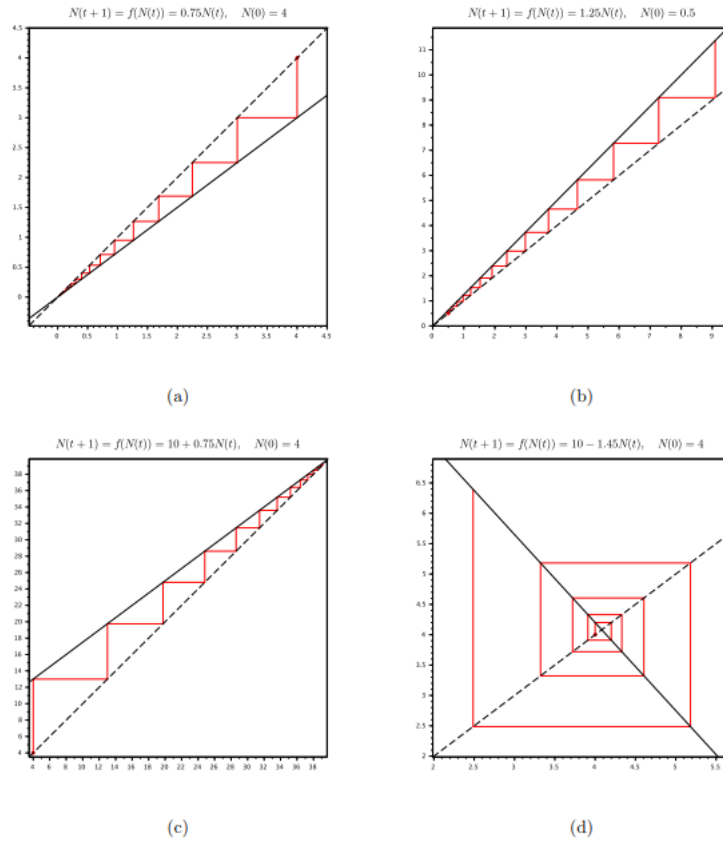


Figure 1: Cobwebbing for a variety of linear models of the form  $N(t+1) = f(N(t))$ . On each plot, the linear function  $f(N)$  is depicted by a solid black line, the diagonal by a dotted black line and the cobweb by a series of red segments. The initial value  $N(0)$  was chosen randomly. It is marked by a red dot on each of the plots.

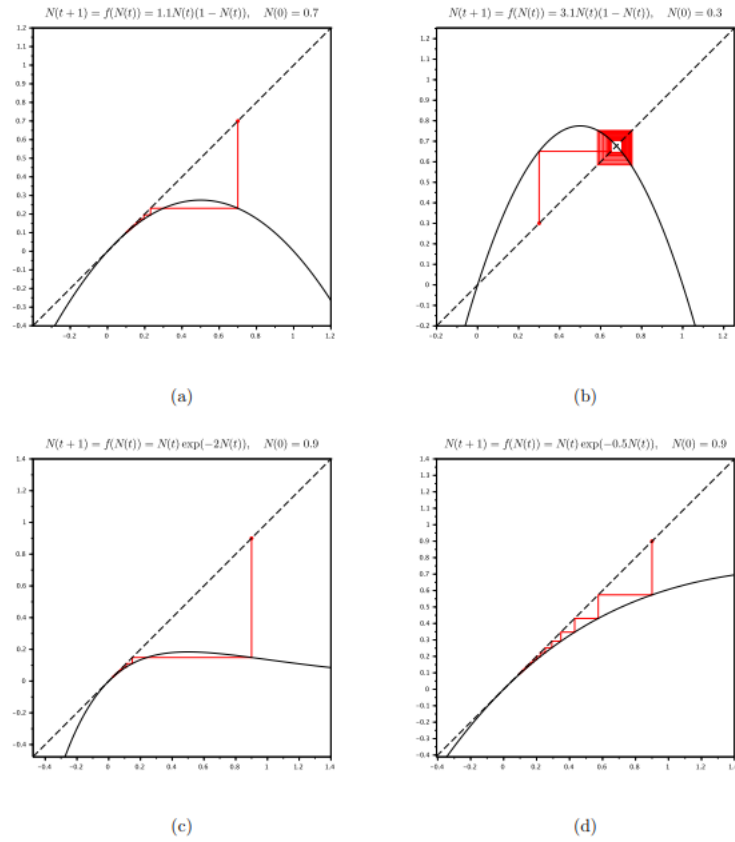


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