MATH 3MB3 FALL 2018 HOMEWORK 1
Due Monday September 24 at 11:59PM

Format your report using some form of word processing software (Word, Latex, OpenOffice, ...), export it to a PDF file and submit it via email to:

- Alexandra Bushby, bushbya@mcmaster.ca if your last name starts with A-G or if you are submitting using R
- Robert White, whitere@mcmaster.ca if your last name starts with a H-Z and you don't plan on using R

together with a file containing the code you used for your computer simulations.

Question 1

We wish to model the population change in a city somewhere in Canada. The census results are obtained each year on January 1st, which we assume to be accurate. On January 1st 2006 there were 500000 people (1/2 a million, can make units easier) living in Hamilton. It is noted that there are three major events each year that change the population.

- At the end of the school year in May, 50000 people leave looking for work elsewhere
- At the start of August the population increases by 20% with new students and people looking for employment
- At the end of December 20000 leave looking for warmer climate

For this question let t be the time in years, starting from $t = 0$, since January 1st 2006. Denote P(t) to be the population of this city (in millions of people) after t years. Note this means that $P(0) = \frac{1}{2}$. Now answer the following in order:

(a) How many people are there on January 1st 2007? In other words, what is P(1)?
(b) What is P(2)?
(c) Construct a recursion equation that predicts $P(t+1)$ given $P(t)$
(d) What kind of model is this? Classify it (univariate or multivariate, ...).
(e) Find an explicit formula for P(t) for all time t.
(f) What is P(18)? Verify the result you get from the explicit formula against that from a numerical calculation of the sequence.

(Note this question requires a graph or perform a computation using your computer)

(g) Does this model have any fixed points? If so find them.

(h) Determine the stability of the fixed point(s).

1. Question 2

The SI model is a basic epidemic model that was mentioned in class. Let S(t) represent the number of people susceptible to a disease, I(t) the number of infected individuals, and N the (time-independent) total population. Now assume that everyone is either susceptible or infected (N = S(t) + I(t)), that susceptible individuals catch the disease from infected individuals through a contact rate , and that infected individuals recover at a rate of . Then one can eliminate I(t) from the equations to find that S(t) is governed by

$S(t+1) = S(t) + (-\beta S(t) + \gamma)(N - S(t))$.

You may think of N, β and γ as known constants.

(a) What kind of model is this? Classify it (univariate or multivariate, ...).

(b) Does this model have any fixed points? If so find them.

(c) Determine the stability of the fixed point(s).

2. Question 3

This question concerns the cubic growth model which is an extension of the logistic model studied in class. It has the usual form

$N(t+1) = f(N(t))$

where $f(N)$ is now a cubic function. More precisely, the recursion equation is

$N(t+1) = N(t)(1 - R(1 - \frac{N(t)}{L})(1 - \frac{N(t)}{K}))$

Here t is time and N the population size. Three constants R, L, and K are visible: R is a reproductive constant, L a constant associated with the Allee effect, and K is the carrying capacity as it was for the logistic model. Lets assume that $R > 0$ and $K > L > 0$. The initial condition $N(0)$ may be assumed to be a known constant. Work out the following questions on paper. Justify your answers.

(a) What kind of model is this? Classify it (univariate or multivariate, ...).

(b) Does this model have any fixed points? If so find them.

(c) Determine the stability of the fixed point(s).

Now use computer software to help answer several more questions. You can check your results here against your findings in parts b) and c). Set R = 0.2, L = 4 and K = 8.

(d) What are the fixed points now?

(e) Run three numerical experiments with initial conditions (N(0)=1, N(0)=5, N(0)=11) and iterate far out enough (say t=20) to see what the long term dynamics of each of these experiments are. What happens to N(t) in these three cases? Do these results agree with b) and c)?