The general formula is $N(t+1) = f(N(t))$, where f is a linear function called the iteration function and t is an integer that usually represents time. Typically, the state variable N is continuous.
We will use this extremely simple model to illustrate our general approach to dynamical modelling, which is as follows:

(1) Solve for equilibria
(2) Determine the stability of found equilibria points
(3) Possibly analyze limiting cases where N is near a boundary
(4) Solve for time-dependent solution or simulate various special cases through time

1. Geometric growth or decay

Linear models are the simplest example of a univariate discrete deterministic model. In this linear case we have that $f(N(t))$ is simply $RN(t)$, though it can also be stated a s$(1+r)N(t)$. This equation is used for (naive) population models or for interest from a bank. Now using our general approach, how do we go about doing step 1)?

$$N(1) = RN(0)$$
$$N(2) = RN(1) = R^2N(0)$$
$$N(t) = R^tN(0)$$

This is an explicit formula for N(t) found through recursion. This formula tells us everything about the models dynamics. Assume that $N(0) > 0$ and $R > 0$. If $R < 1$, then $N(t) \to 0$ for $t \to \infty$. However it's important to note that N(t) will never reach 0 in a finite amount of time, unless $N(0) = 0$. If $R = 1$ then $N(t) = N(0)$. If $R > 1$ then $N(t) \to \infty$ for $t \to \infty$.

If we can find a value N^* where $N^* = f(N^*)$, then N^* is called an equilibrium or fixed point of the system. The reason for this is even after doing another iteration of N^*, you still stay at the same point or value where you started, namely N^*. Therefore N^* is fixed for all iterations and all time. Equilibrium are found by setting $N(t+1) =$

$N(t) = N^*$ then solving $N^* = f(N^*)$. In the case of our system we get $N^* = RN^*$ which becomes $N^*(R-1) = 0$. So either R=1 and all points are fixed points, or the only fixed point is $N^* = 0$. In the case of a non linear model, trying to find this equilibrium will always be harder.

Now stability of the equilibrium deals with what happens around the fixed point (perturbations around the fixed point). Consider if we displace the population away from N^* by δ, where $\delta << 1$. If $|R| < 1$ then N will return to N^*. If $|R| > 1$ then N will go off to infinity. Therefore we conclude that the fixed point $N^* = 0$ is stable iff $|R| < 1$. There happens to be a general condition for the stability of a fixed point of a first-order recurrence equation $N(t+1) = f(N(t))$, namely that the fixed point if stable if $|f'(N^*)| < 1$.

2. Affine Models

Now suppose that we add or subtract a term to the first-order recurrence equation. For example, N(t+1)=a+RN(t). Due to the fact that this model has the additional a term added on it makes it called an affine model instead of just simply a linear model. Working out the recursion on paper yields:
$N(1) = a + RN(0)$
$N(2) = a + RN(1) = a + R(a + R(N(0))) = a(1+R) + R^2N(0)$
$N(3) = a+RN(2) = a+R(a(1+R)+R^2N(0)) = a(1+R+R^2)+R^3N(0)$
$N(t) = a\frac{1-R^t}{1-R} + R^tN(0)$
or by regrouping can be written as: $N(t) = \frac{a}{1-R} + (N(0) - \frac{a}{1-R})R^t$
Which then makes the fixed point $N^t = \frac{a}{1-R}$. A simpler way of finding this would be from solving the equation $N^* = a + RN^*$. Since $f(N) = a + RN$, $f'(N) = R$, then by the stability we had from linear, the fixed point is stable if $|R| < 1$ and unstable if $|R| \geq 1$. Why does this case have $|R|$=1 being unstable? If $0 < R < 1$ then this is known as a bucket model where a is the supply rate and $\frac{1}{1-R}$ is the average residency time.

3. Multiple Lags

What happens in the case where N(t) not only depends on N(t-1) but N(t-2) as well? For example lets look at $N(t+2) = 3N(t+1) - 2N(t)$. The first thing to do is to look at the homogeneous equation, which for this example is $N(t+2) = 3N(t+1) - 2N(t)$, where N(0)=1 and N(1)=2. Now we plug in $N(t) = C\lambda^t$, giving us $C\lambda^{t+2} = 3C\lambda^{t+1} - 2C\lambda^t$, which after simplification becomes $\lambda^2 - 3\lambda + 2 = 0$, which has solutions of $\lambda = 1$ and $\lambda = 2$. Which then means the general solution of the homogenous equation is $N(t) = C_1 1^t + C_2 2^t$. Trying to find the general solution to the original system, we stick in $N(0) = 1$ and $t = 0$ to get the equation $C_1 + C_2 = 1$. Then we stick in $N(1) = 2$ and $t = 1$ to get the equation $C_1 + 2C_2 = 2$. Combining these two gives up $C_1 = 0$ and $C_2 = 1$. Therefore the solution is $N(t) = 2^t$. Now to double check our work.

- $N(0) = 2^0 = 1$
- $N(1) = 2^1 = 2$
- $N(t+2) - 3N(t+1) + 2N(t) = 2^{n+2} - 32^{n+1} + 2^{n+1} = 2^{n+2} - 22^{n+1} = 0$