Course: Math 3MB3 Midterm
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Honor Statement: By signing below I confirm that I have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator, notes, computer, or cell phone, any of which will result in an exam mark of 0. Furthermore, I agree not to discuss this exam with anyone until the exam testing period is over

Signature:
Date:

1. Question 1

$N(t+1) = N(t)^2 - 2N(t) - 4$
a) Classify this Model
b) Find the fixed point(s) of this model
c) Determine the stability of the fixed point(s).

Question 1 is out of 9 marks
a) Nonlinear Univariate Discrete Deterministic
1 mark
b) Let $N^* = N(t) = N(t+1)$ then
$N^* = (N^*)^2 - 2N^* - 4$
$(N^*)^2 - 3N^* - 4 = 0$
$(N^* - 4)(N^* + 1) = 0$
$N^* = -1$ or $N^* = 4$
Therefore the fixed points are -1 and 4
1 mark for putting in N^*
1 mark for rearranging
1 mark for fixed points
c) Let $f(N) = N(t)^2 - 2N(t) - 4$, then
$f'(N) = 2N(t) - 2$
$f'(-1) = -4$
$f'(4) = 6$
Since $|f'(-1)| > 1$, the fixed point -1 is unstable.
Since $|f'(4)| > 1$, the fixed point 4 is unstable.
1 mark for correct derivative
1 mark for sticking in both fixed points correctely
1 mark for checking derivative of both fixed points in abs vs 1
1 mark for each correct conclusion of the fixed points (2 total)

2. Question 2

$x' = 2 - x$
a) Classify the model
b) Find the fixed point(s) of this model
c) Determine the stability of the fixed point(s)
d) Find the explicit solution of this model

Question 2 is out of 12 marks
a) linear Univariate Continuous Deterministic
1 mark
b) Let $x' = 0$, and $x^* = x$, then $0 = 2 - x^*$
$2 = x^*$ Therefore the fixed point is $x^* = 2$
1 mark for sticking in x^*
1 mark for fixed point
c) Let $f(x) = 2 - x$ then
$f'(x) = -1$
$f'(2) = -1$
Since $f'(2) < 0$, the fixed point 2 is unstable.
1 mark for correct derivative
1 mark for checking $f'(2)$
1 mark for checking derivative of fixed point < 0
1 mark for correct conclusion
d) $\frac{dx}{2-x} = dt$
$\frac{dx}{x-2} = -dt$
$\int \frac{1}{x-2} dx = -\int 1 dt$
$ln(x-2) = -t + C_1$
$x - 2 = C_2 e^{-t}$
$x = C_2 e^{-t} + 2$
at $t = 0$, $x(t) = x(0) = C_2 + 2$
$C_2 = x(0) - 2$
$x(t) = (x(0) - 2)e^{-t} + 2$
1 mark for separation of variables
1 mark for integration
1 mark for using the exponential
1 mark for solving for C_2 in terms of x(0)
1 mark for final answer

3. Question 3

$N(t+2) = 2N(t+1) + 3N(t)$
a) Classify the model
b) Determine the fixed point(s)
c) Find the explicit solution of the model
d) Determine the stability of the fixed point(s)

10 marks for this question
a) Linear Univariate Discrete Deterministic
1 mark
b) Let $N(t) = N(t+1) = N(t+2) = N^*$ then
$N^* = 2N^* + 3N^*$
$0 = 4N^*$
$N^* = 0$
Therefore $N^* = 0$ is the one fixed point
1 mark for setup
1 mark for finishing
c) Let $N(t) = C\lambda^t$ then
$\lambda^2 - 2\lambda - 3 = 0$
$(\lambda - 3)(\lambda + 1) = 0$
Therefore $\lambda = -1$ or 3
$N(t) = C_1(-1)^t + C_2 3^t$
1 mark for the first line
1 marks for solving for both λ
1 mark for explicitly stating N(t)
d)Method 1:
limit as t approaches ∞ of $N(t) = \infty$ Therefore the fixed point 0 is unstable
Method 2:
Let $f = 5N$ then $f'(N) = 5$ and $|f'(0)| = 5 > 1$ therefore the fixed point 0 is unstable
Method 3:
$\lambda_1 = 3 > 1$ therefore unstable
1 Mark for taking derivative
1 Mark for sticking in fixed point
1 mark for checking derivative of the fixed point in absolute value vs 1
1 mark for correct conclusion of the fixed point

4. Question 4

$$\begin{bmatrix} J(t+1) \\ A(t+1) \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 6 & 6 \end{bmatrix} \begin{bmatrix} J(t) \\ A(t) \end{bmatrix}$$

a) Classify the model
b) Find the fixed point(s)
c) Determine the stability of the fixed point(s)

9 Marks for this question a) Linear Multivariate Discrete Deterministic
1 mark
b) $J(t+1) = 3J(t) + A(t)$
$A(t+1) = 6J(t) + 6A(t)$
$LetA(t) = A(t+1) = A^*$ and $LetJ(t) = J(t+1) = J^*$ then
$0 = 2J^* + A^*$
$0 = 6J^* + 5A^*$
Take second equation -3 of the first equation and get:
$0 = 2A^*$ Therefore $A^* = 0$ then using either equation $J^* = 0$
Therefore the fixed point is (0,0)
1 Mark for setting up the two equations correctly
1 Mark for sticking in the fixed points
2 Mark for solving
c) Trace = 9, Determinate = 12 therefore
$\lambda^2 - 9\lambda + 12 = 0$
$\lambda = \frac{9 \pm \sqrt{9^2 - 4*12}}{2}$
Therefore the larger eigenvalue is $> \frac{9}{2} > 1$
Therefore the fixed point is unstable
1 Mark for getting the correct characteristic equation
1 Mark for finding eigenvalues
1 Mark for comparing with 1
1 Mark for conclusion