1. Hartman Grobman Theorem

Hartman Grobman basically states that our original system, mimics the linearization(Jacobian) as long as the eigenvalues aren't purely imaginary. If we start with a two by two matrix:
From what we had before, let $T = Trace(A)$ and $\Delta = Det(A)$ then we have that $\lambda^2 - T\lambda + D = 0$
Lets look at the following example:
$\frac{dx}{dt} = -y - x(x^2 + y^2)$
$\frac{dy}{dt} = x - y(x^2 + y^2)$
Then the Jacobian of this system is:

$$\begin{bmatrix} -3x^2 - y^2 & -1 - 2xy \\ 1 - 2xy & -x^2 - 3y^2 \end{bmatrix}$$

Given that the only equilibria of this system is (0,0) the Jacobian at this equilibria is:

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

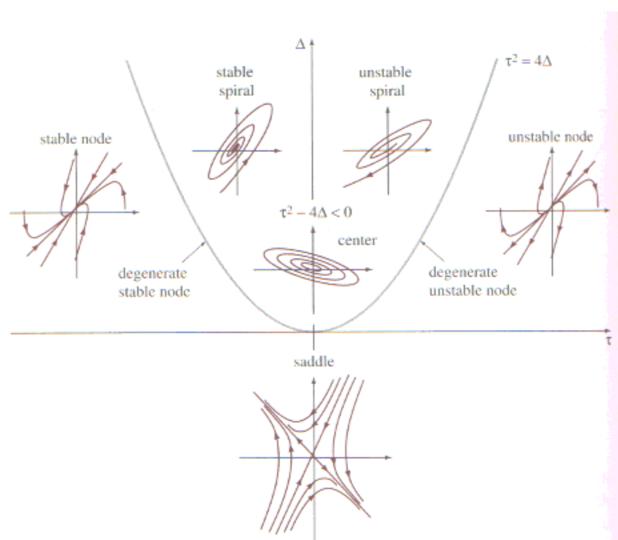

Figure 1. Note that the circles on the upper y axis are actually a mistake. Image from http://minitorn.tlu.ee/ jaagup/uk/dynsys/ds2/nonlinear/local/local.html

Which gives a trace of 0 and a determinant of 1. If we tried to use the above graph it says we should get a center.

If instead we change to polar coordinates. As in $x = rcos(\theta)$ and $y = rsin(theta)$ and $x^2 + y^2 = r^2$ and $\theta = arctan(\frac{y}{x})$ we get the following:
$\theta = arctan(\frac{y}{x})$
If we differentiate both sides of this we get the following:
$\frac{d\theta}{dt} = \frac{1}{1+(\frac{y}{x})^2}\frac{y'x-x'y}{x^2}$
$= \frac{y'x-x'y}{x^2+y^2}$
$= \frac{y'x-x'y}{r^2}$
$= \frac{(x-y(x^2+y^2))x-y(-y-x(x^2+y^2))}{r^2}$
$= \frac{r^2}{r^2} = 1$

$r^2 = x^2 + y^2$
If we differentiate both sides of this we get the following:
$2rr' = 2xx' + 2yy'$
$r' = \frac{-2xy-2x^2(r^2)+2xy-2y^2(r^2)}{2r}$
$r' = \frac{-2r^4}{2r} = -r^3$
We can now see that non-zero trajectories will decay towards (0,0) and (0,0) becomes a stable spiral, not a center. So be careful if your eigenvalues had zero real part.

2. Routh-Hurwitz stability criterion

Looking at the characteristic polynomial for a 2 by 2 case we have $\lambda^2 + a_1\lambda + a_0 = \lambda^2 - T\lambda + D = 0$. We have stability of the fixed point if both a_0 and a_1 are positive. Which translates to the trace has to be negative, and the determinate has to be positive.
Looking at the characteristic polynomial for a 3 by 3 case we have $\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0$. We have stability of the fixed point if both a_0, a_1, a_2 are positive and $a_2a_1 > a_0$.
This goes for higher and higher matrices but gets more and more complicated.

3. Gershgorin Circle Theorem

Let A be a complex n by n matrix, with entries a_{ij} where $i, j \in 1, 2, \cdots, n$. Let $R_i = \sum_{j\neq i} |a_{ij}|$, and $D(a_{ii}, R_i)$ denote the closed disc centered at a_ii with radius R_i, then every eigenvalue of A lies within at least one of the discs $D(a_{ii}, R_i)$.

This theorem seems rather wordy, but is easy to apply. Lets look at the following 4 by 4 example:

$$J(x_1^*, x_2^*, x_3^*, x_4^*) \begin{bmatrix} -5 & 1 & 1 & 1 \\ -1 & -4 & -2 & 3 \\ -1 & 1 & -7 & 1 \\ 1 & 1 & 3 & -6 \end{bmatrix}$$

Looking at the rows we get the following discs (D(-5,3), D(-4,6), D(-7,3), D(-6,5)) which doesn't tell us the stability or if we can even linearize since the disc D(-4,6) allows positive eigenvalues. However if we instead look at the columns we get the following discs (D(-5,3), D(-4,3), D(-7,6), D(-6,5)). Now we have that no discs allow positive numbers or 0, so we can linearize, and the fixed point will be stable since all eigenvalues have a real part less than 0. This method is quite useful for when the matrices are at least 3 by 3, this way the eigenvalues don't need to be calculated for stability, assuming that you get only negative discs.